Search for stochastic gravitational waves with the SPring-8 storage ring

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[2]Japan Synchrotron Radiation Research Institute, SPring-8, Japan

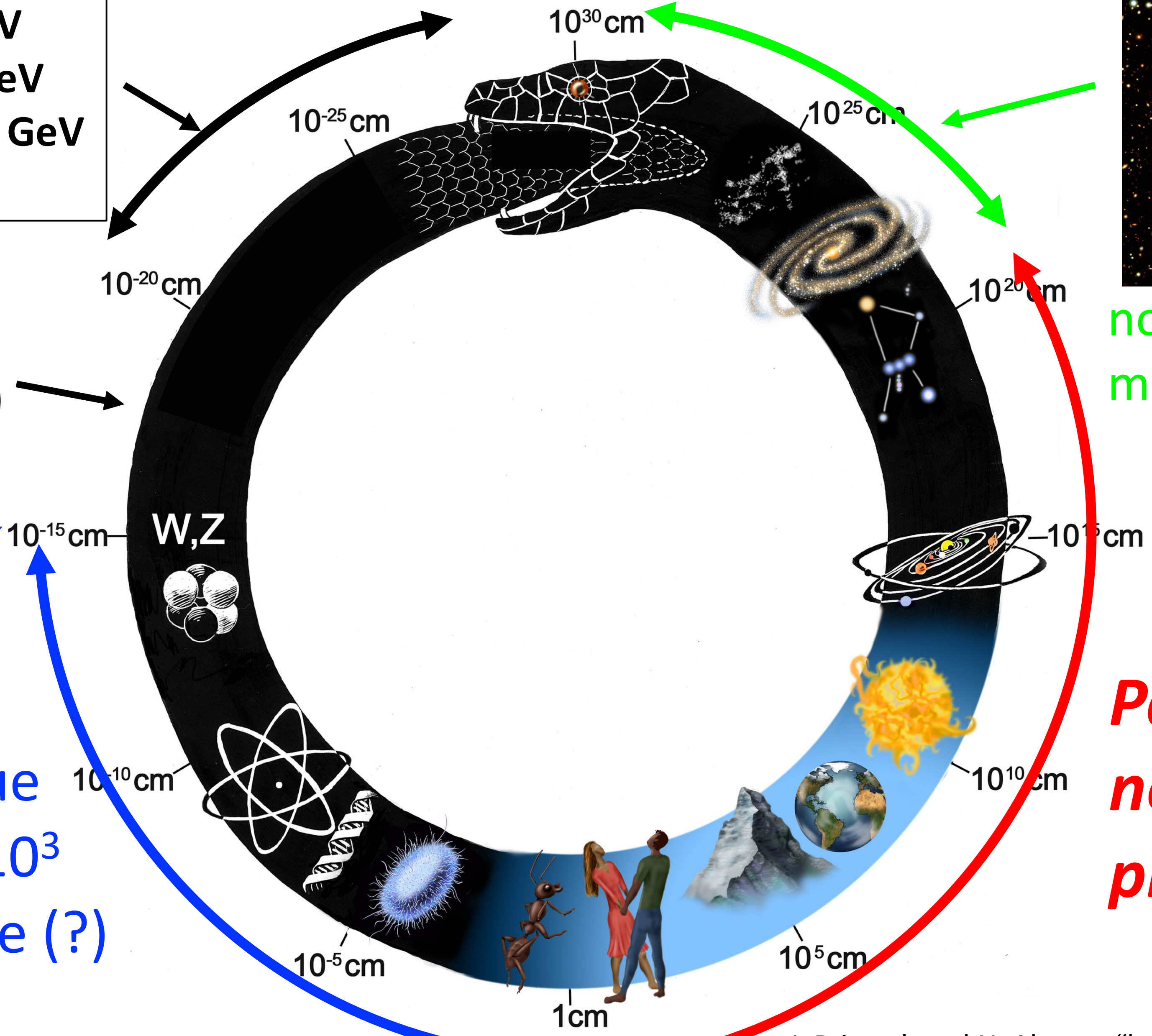
Les nouvelle physiques sont où?

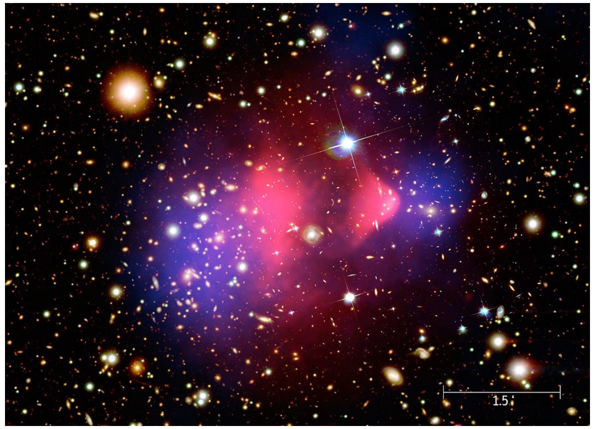
Quantum gravity 10^{19} GeV
Inflation scale < 2×10^{16} GeV
Majorana neutrino $10^{9\text{-}12}$ GeV
Axion PQ scale 10^{10} GeV

(Supersymmetry?)
(Higgs composite?)

Le boson de Higgs 10^3 GeV

nouvelle physique chaque facteur 10^3
→ Exceptionnelle (?)



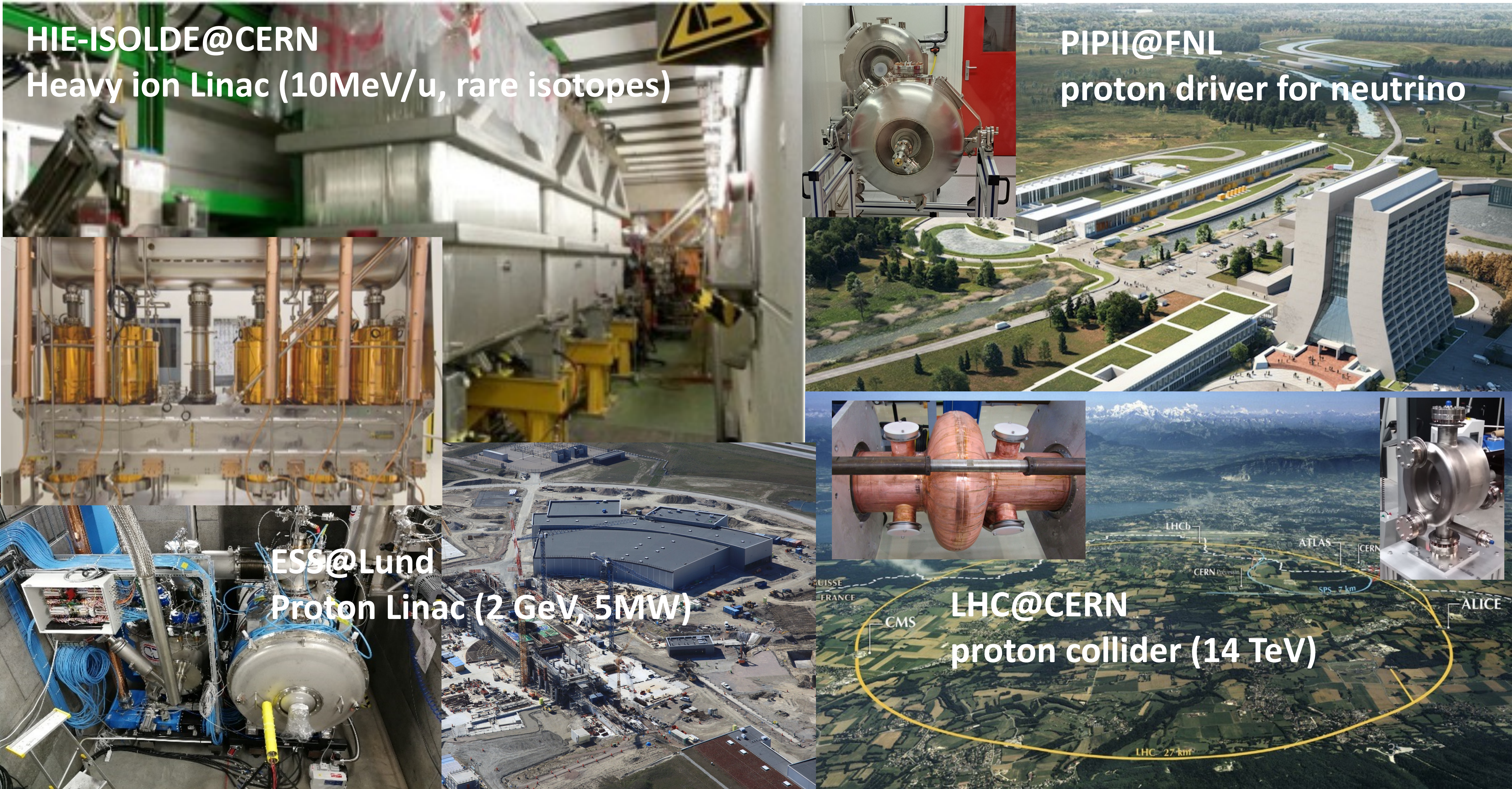


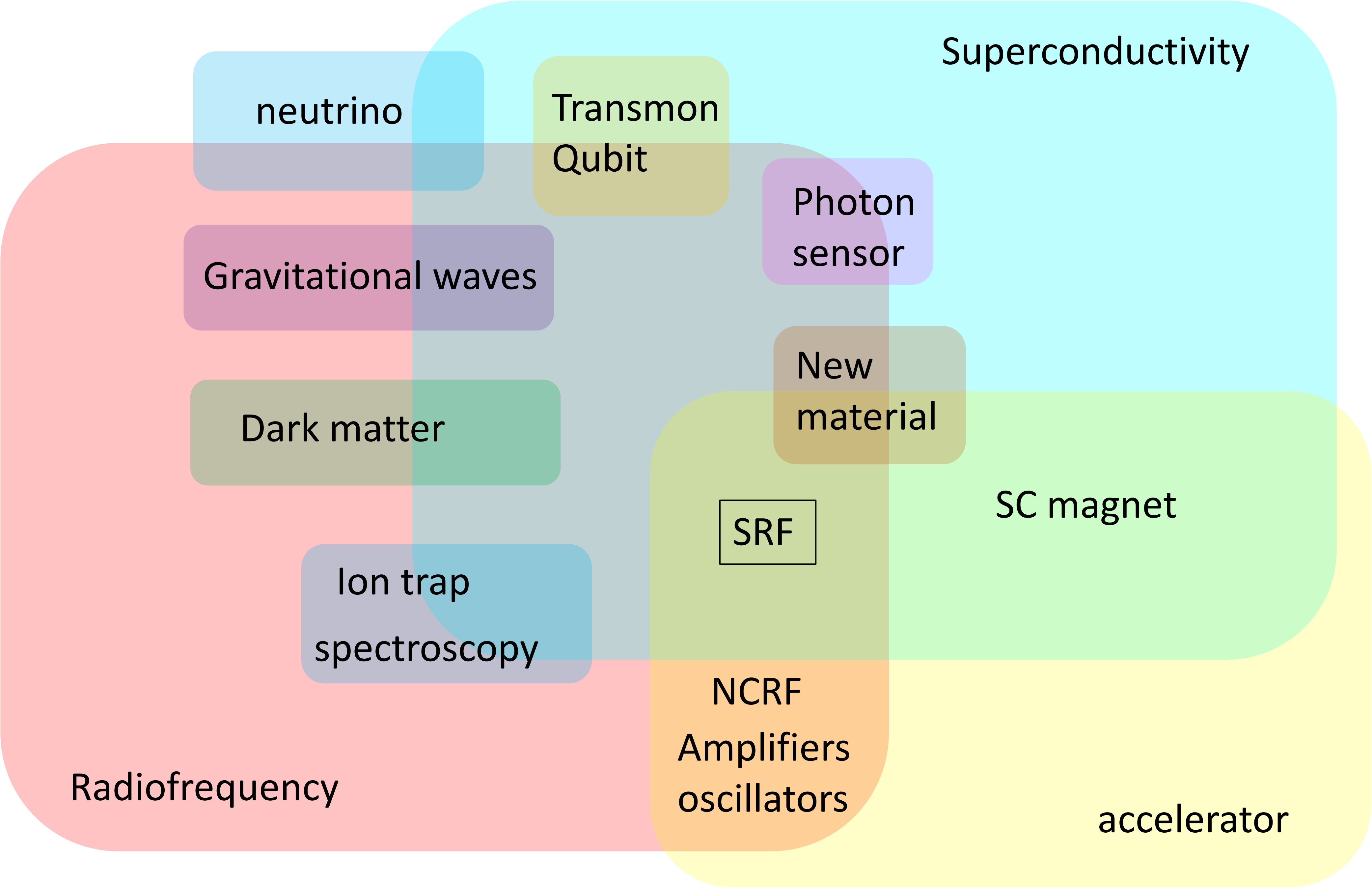
nouvelle physique: matière noire après 10^{20}

Pas de nouvelle physique

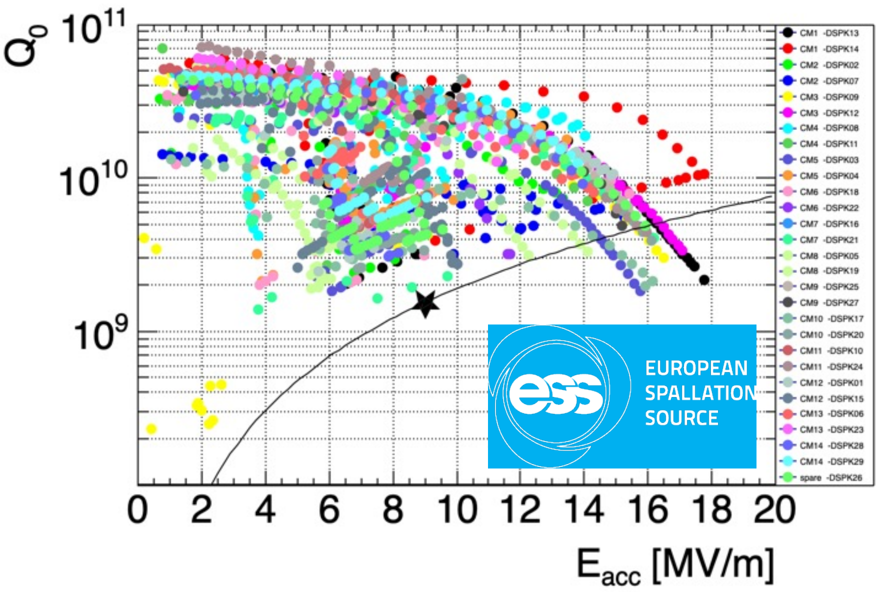
J. Primack and N. Abrams,"http://www.viewfromthecenter.com"

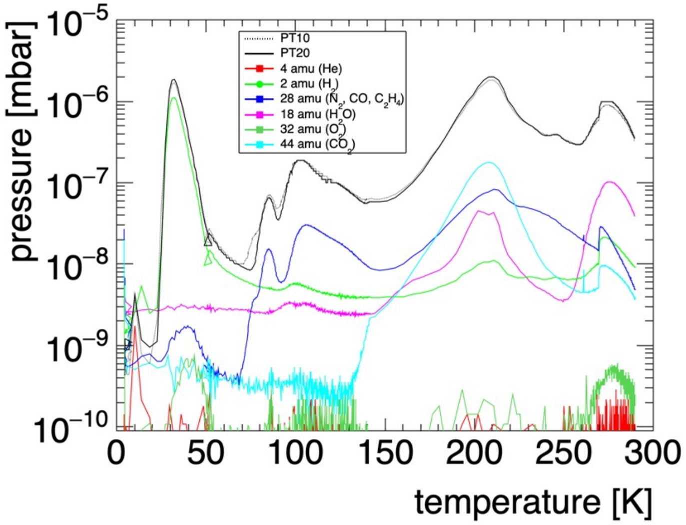
Self introduction: accelerator as infrastructure for science

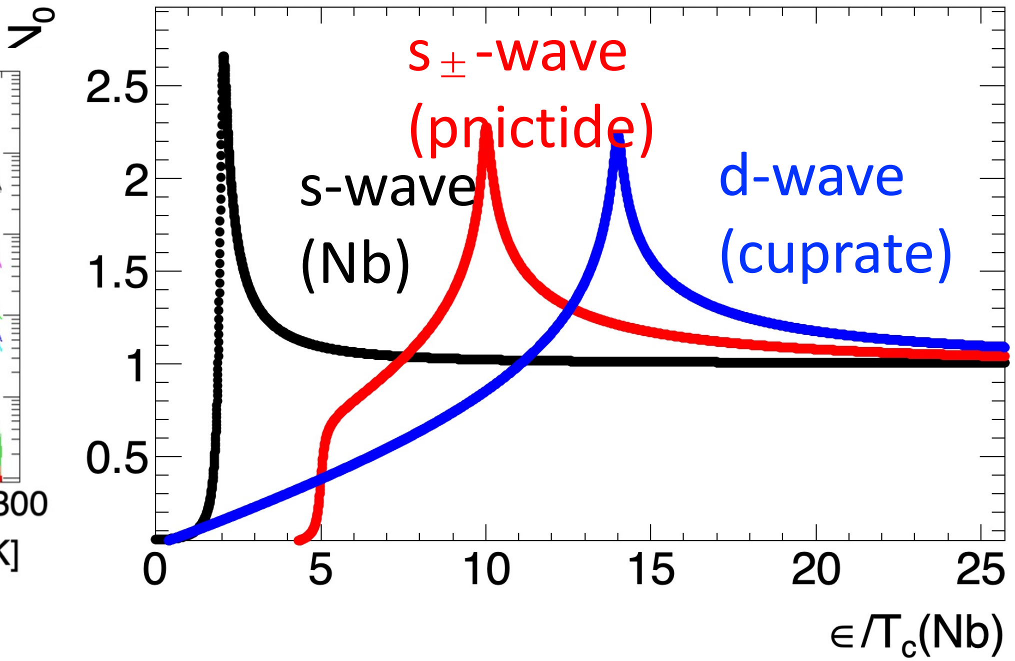


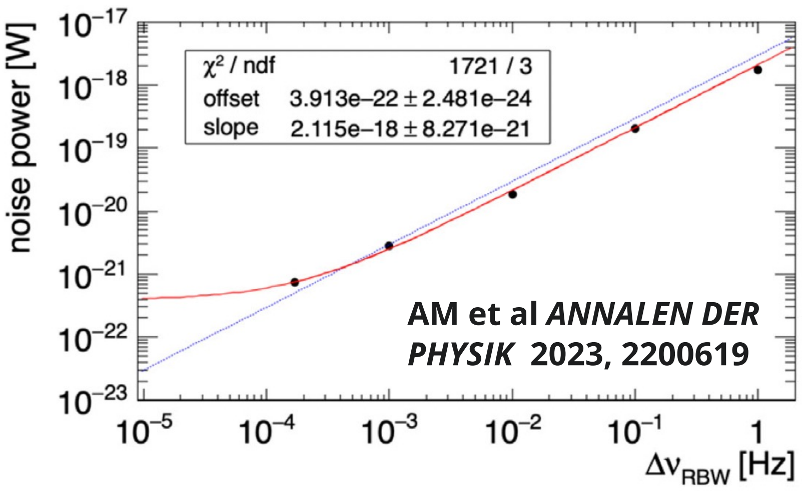

Superconductivity
neutrino
Transmon
Qubit
Photon
sensor
Gravitational waves
New
material
Dark matter
SC magnet
SRF
Ion trap
spectroscopy
NCRF
Amplifiers
oscillators
Radiofrequency
accelerator

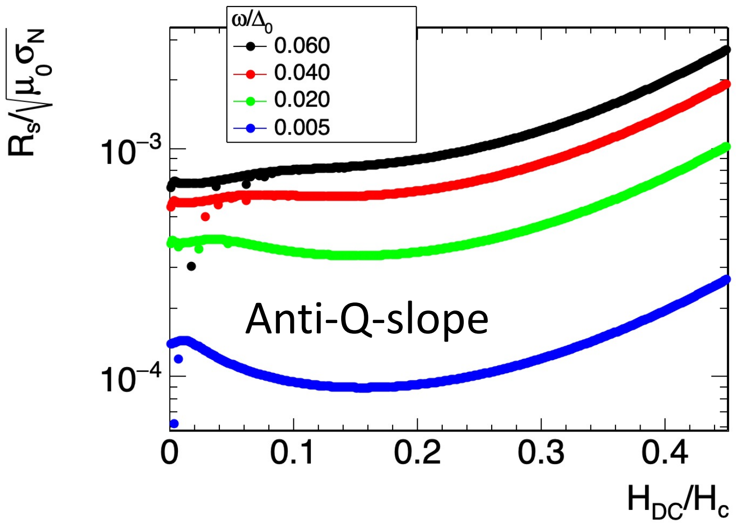
Advertisement: recent studies (→afternoon)

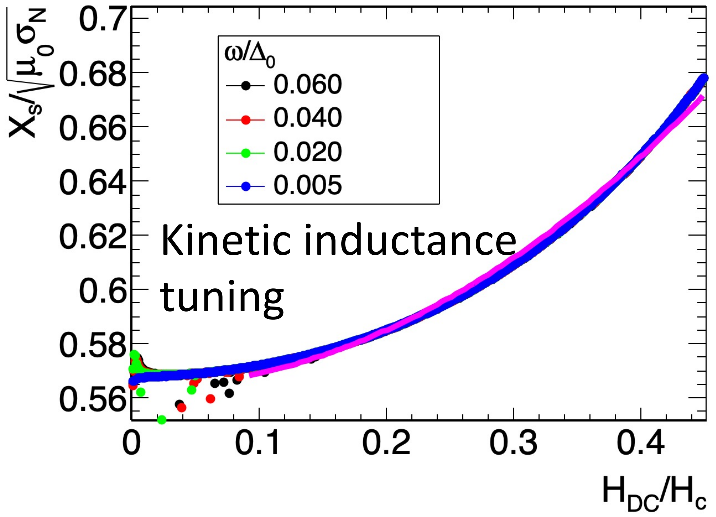


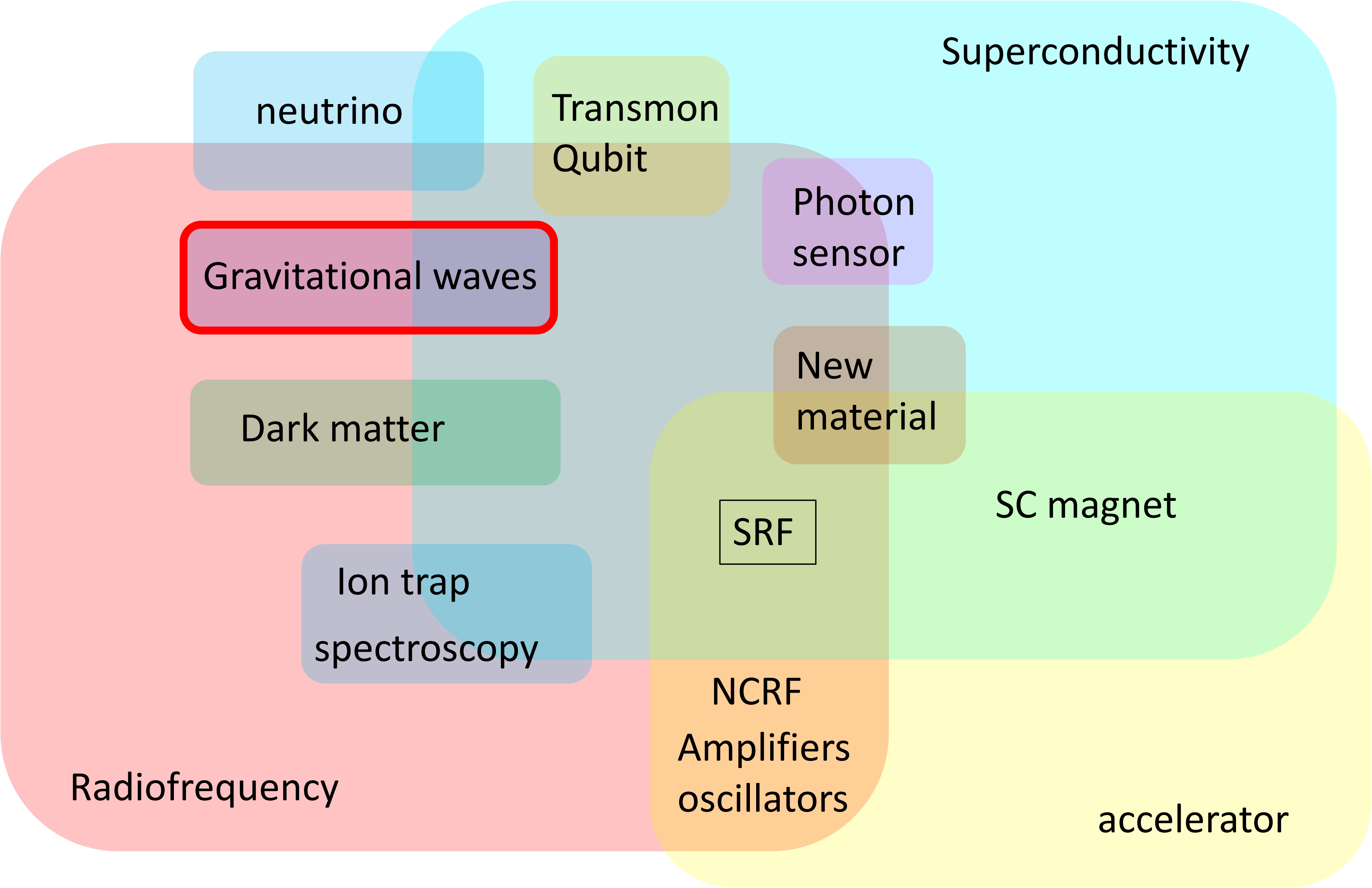











Superconductivity
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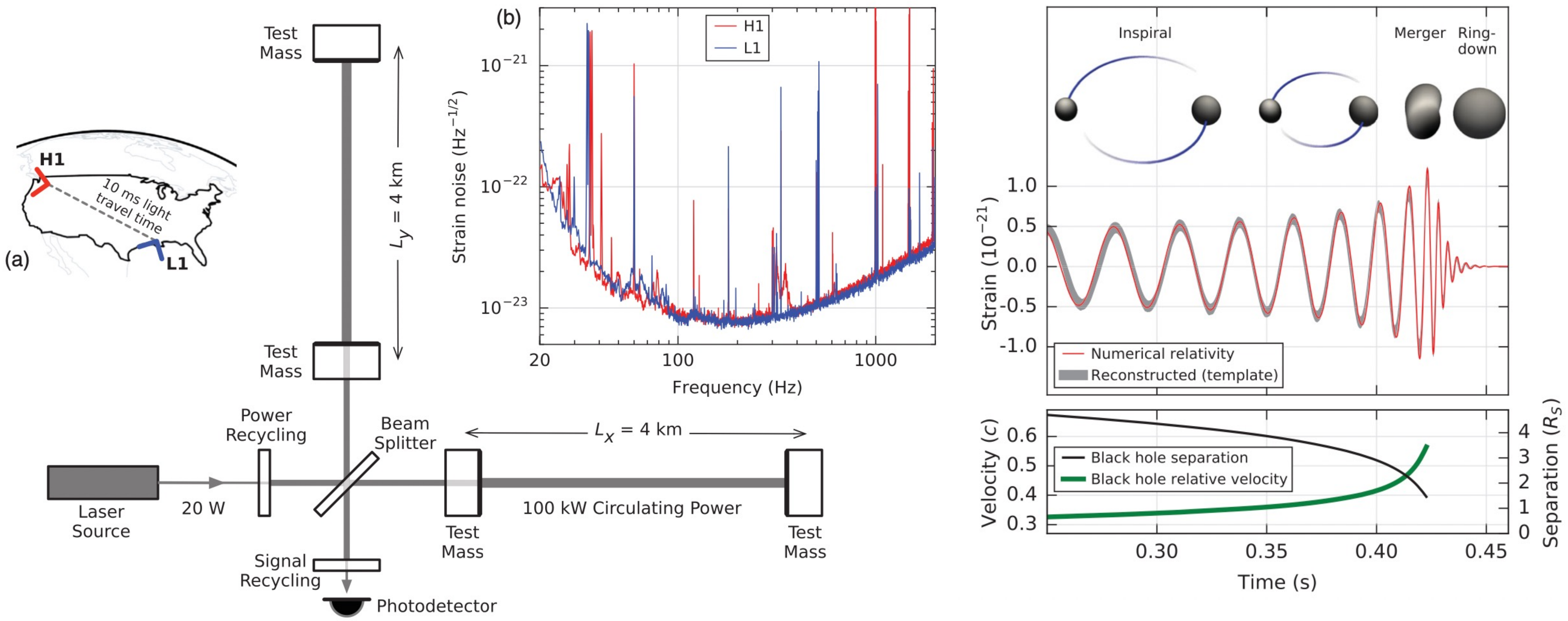
Introduction

Laser GW interferometer

PRL **116**, 061102 (2016)



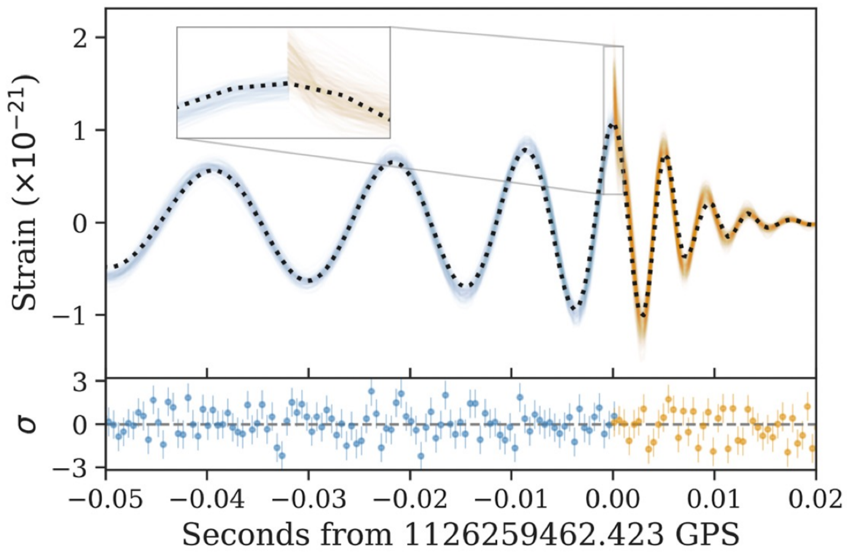
Surface area increase → Bekenstein entropy?

PHYSICAL REVIEW LETTERS **127,** 011103 (2021)

Editors' Suggestion **Featured in Physics**

Testing the Black-Hole Area Law with GW150914

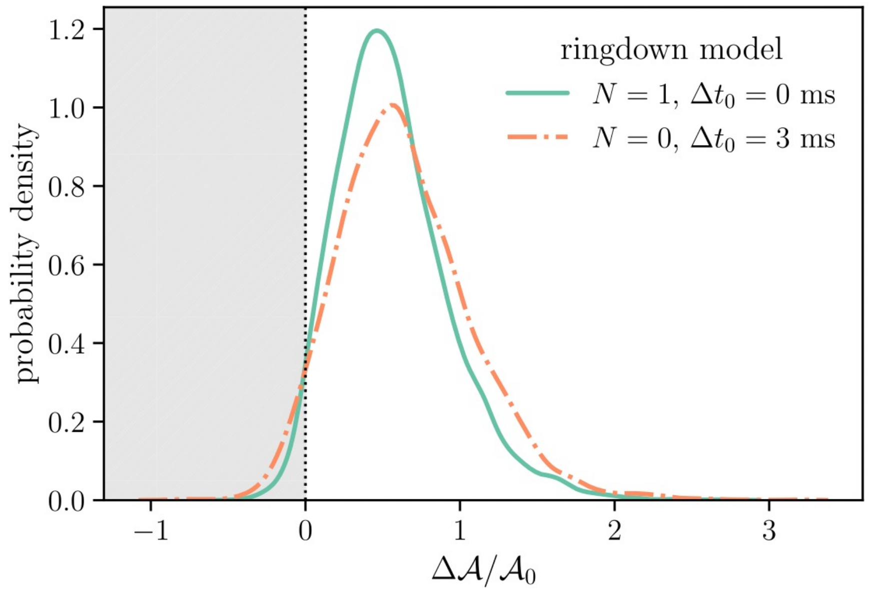
Maximiliano Isi,[1,*] Will M. Farr,[2,3,†] Matthew Giesler,[4] Mark A. Scheel,[5] and Saul A. Teukolsky[4,5]
[1]*LIGO Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Center for Computational Astrophysics, Flatiron Institute, 162 5th Ave, New York, New York 10010, USA*
[3]*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA*
[4]*Cornell Center for Astrophysics and Planetary Science, Cornell University, Ithaca, New York 14853, USA*
[5]*TAPIR, Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA*



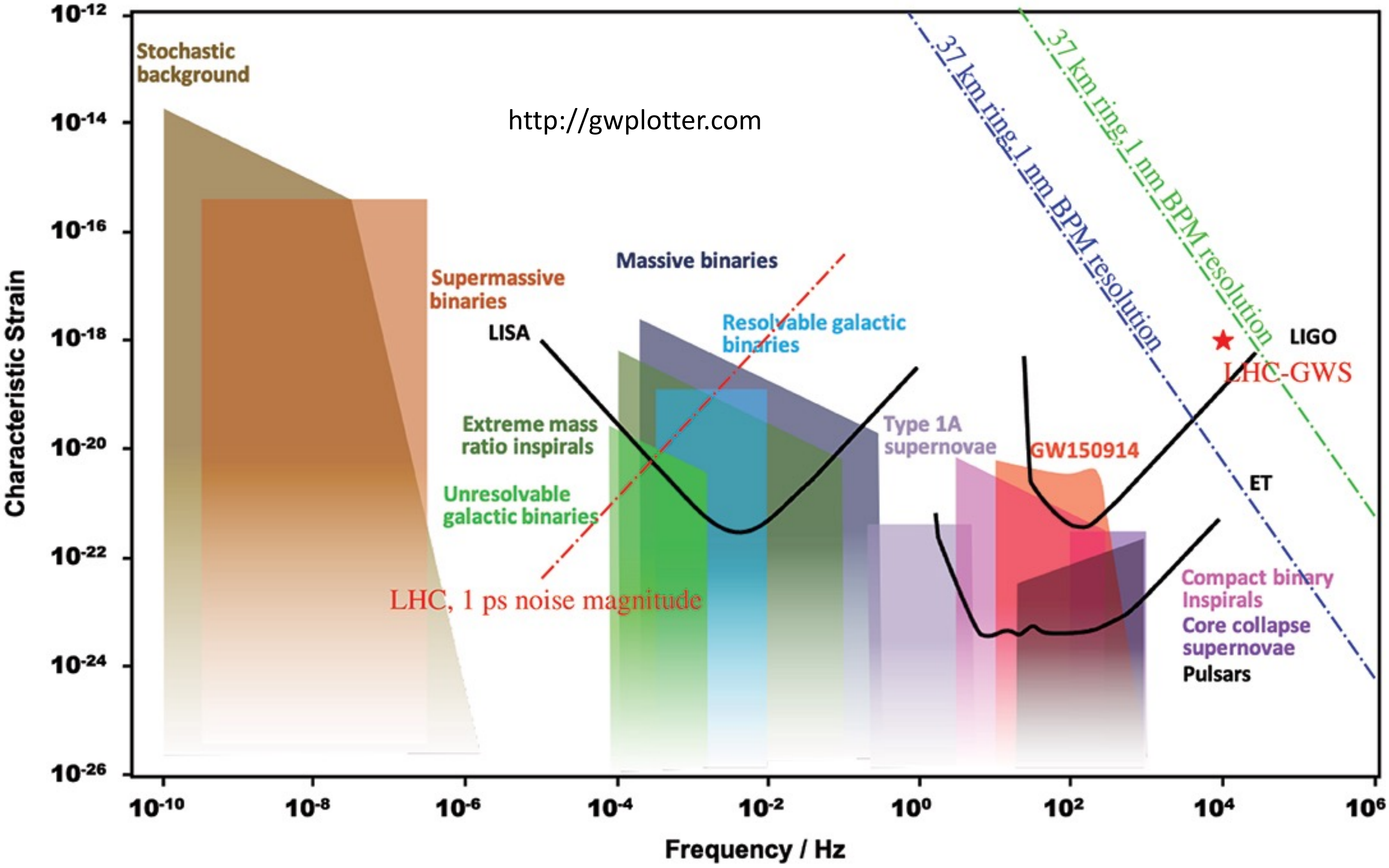
Gravitational Radiation from Colliding Black Holes

S. W. Hawking
Institute of Theoretical Astronomy, University of Cambridge, Cambridge, England
(Received 11 March 1971)

It is shown that there is an upper bound to the energy of the gravitational radiation emitted when one collapsed object captures another. In the case of two objects with equal masses m and zero intrinsic angular momenta, this upper bound is $(2-\sqrt{2})m$.



Exciting opportunities in Blackhole physics



Stochastic background
http://gwplotter.com
37 km ring,1 nm BPM resolution
37 km ring,1 nm BPM resolution
Supermassive binaries
Massive binaries
LISA
Resolvable galactic binaries
LIGO
LHC-GWS
Extreme mass ratio inspirals
Type 1A supernovae
GW150914
Unresolvable galactic binaries
ET
LHC, 1 ps noise magnitude
Compact binary Inspirals
Core collapse supernovae
Pulsars
Characteristic Strain
Frequency / Hz

Two Phenomena to address high frequency GW

The Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

can be expanded to the linear order with small strain h

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Mechanical deformation of a cavity wall

$$\frac{d^2x}{dt^2} = -\frac{1}{2}\frac{d^2h_{xx}}{dt^2}x + \frac{1}{2}\frac{d^2h_{xx}}{dt^2}y$$

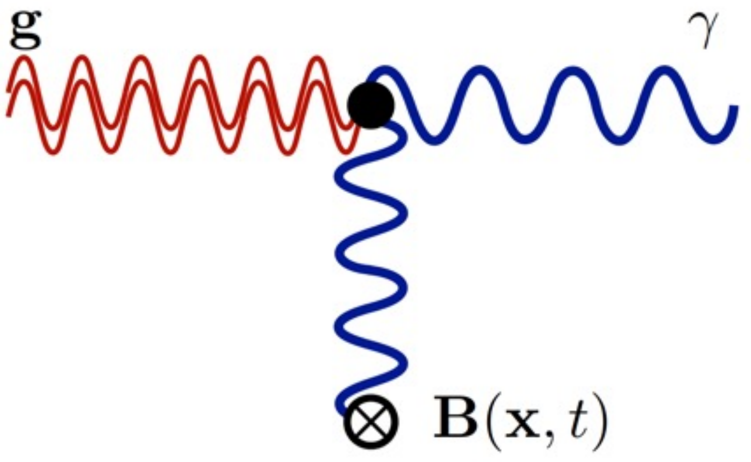
$$\frac{d^2y}{dt^2} = \frac{1}{2}\frac{d^2h_{xx}}{dt^2}x + \frac{1}{2}\frac{d^2h_{xx}}{dt^2}y$$

arXiv:gr-qc/0502054

Coupling to microwaves under static B

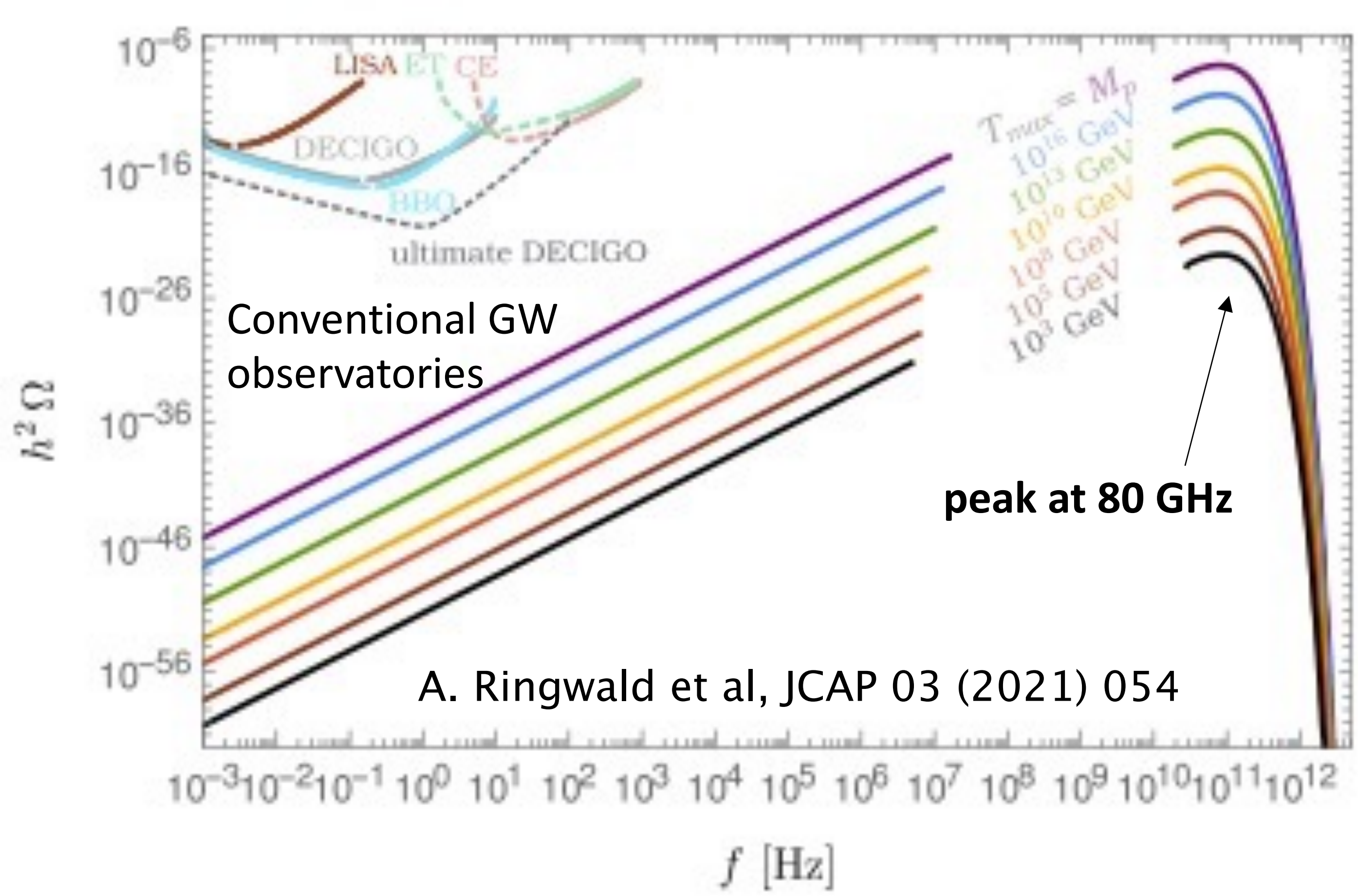
$$\Box h_{\mu\nu} = -16\pi T_{\mu\nu}$$

$$4\pi T_{\mu\nu} = F_{\mu\alpha}F_{\nu}{}^{\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}F^{\alpha\beta}$$

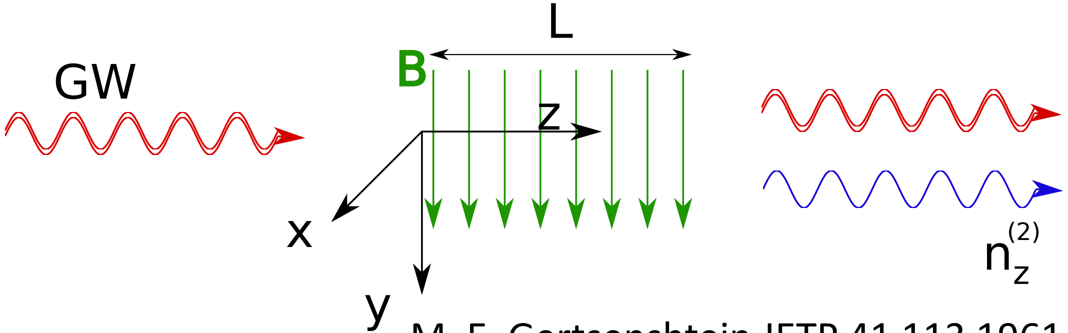


M. E. Gertsenshtein JETP 41 113 1961

High-frequency gravitational waves produced in the thermal plasma in the early universe

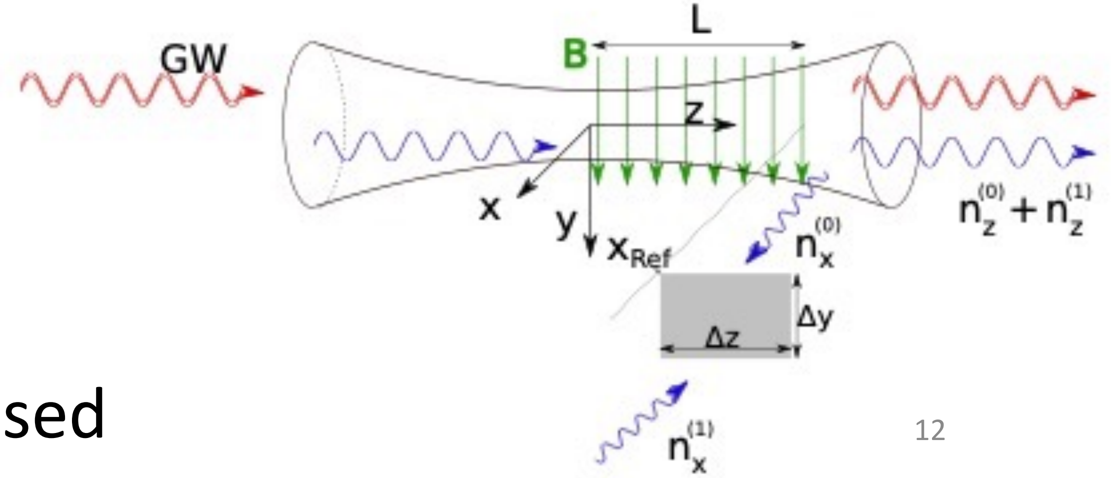


GW couples to photons of the same frequency under static magnetic field

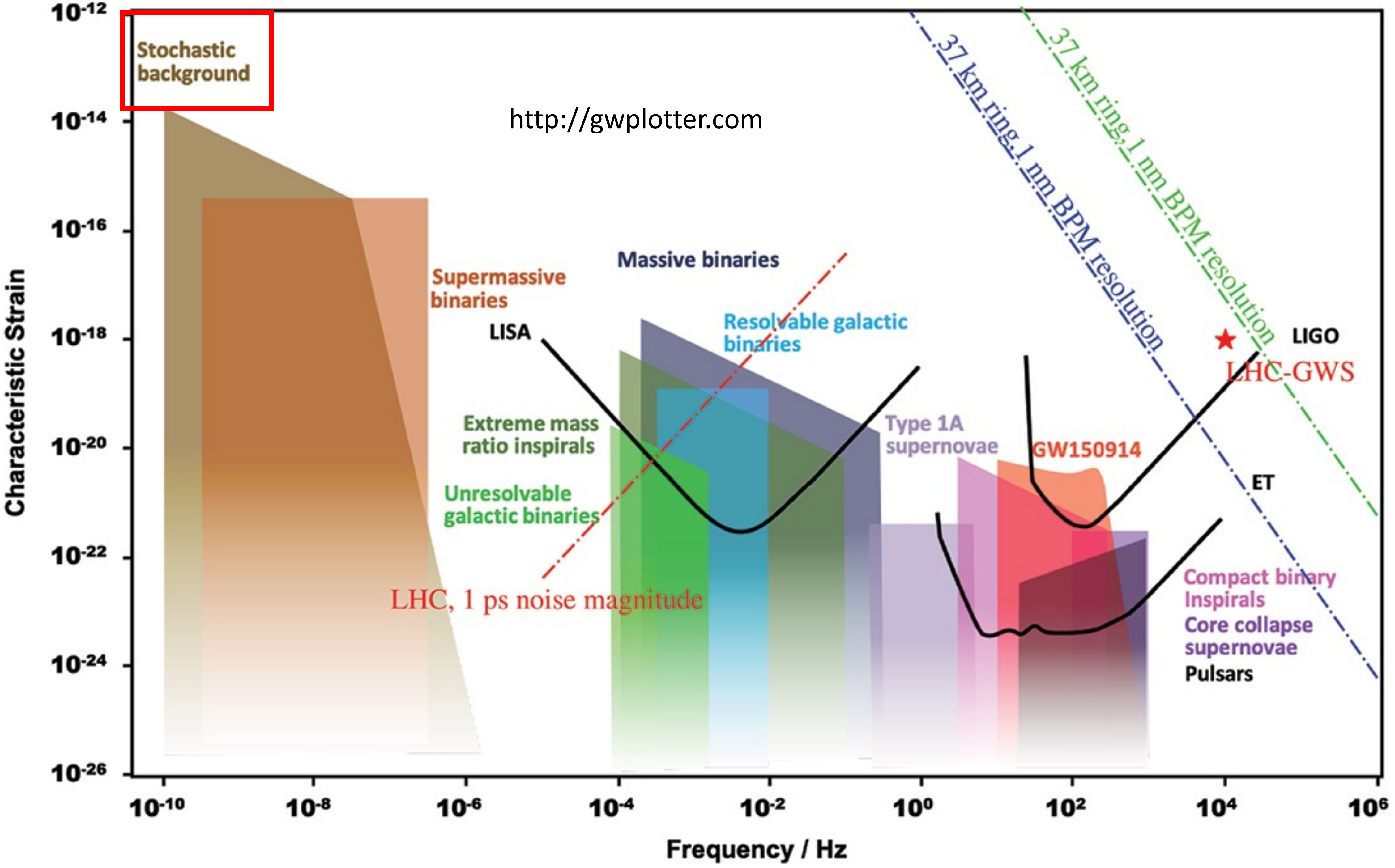


M. E. Gertsenshtein JETP 41 113 1961

High-power 80GHz gaussian beam may enhance the signal level



- Peak at high frequency 80 GHz
- Microwave experiments have been proposed



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Characteristic Strain
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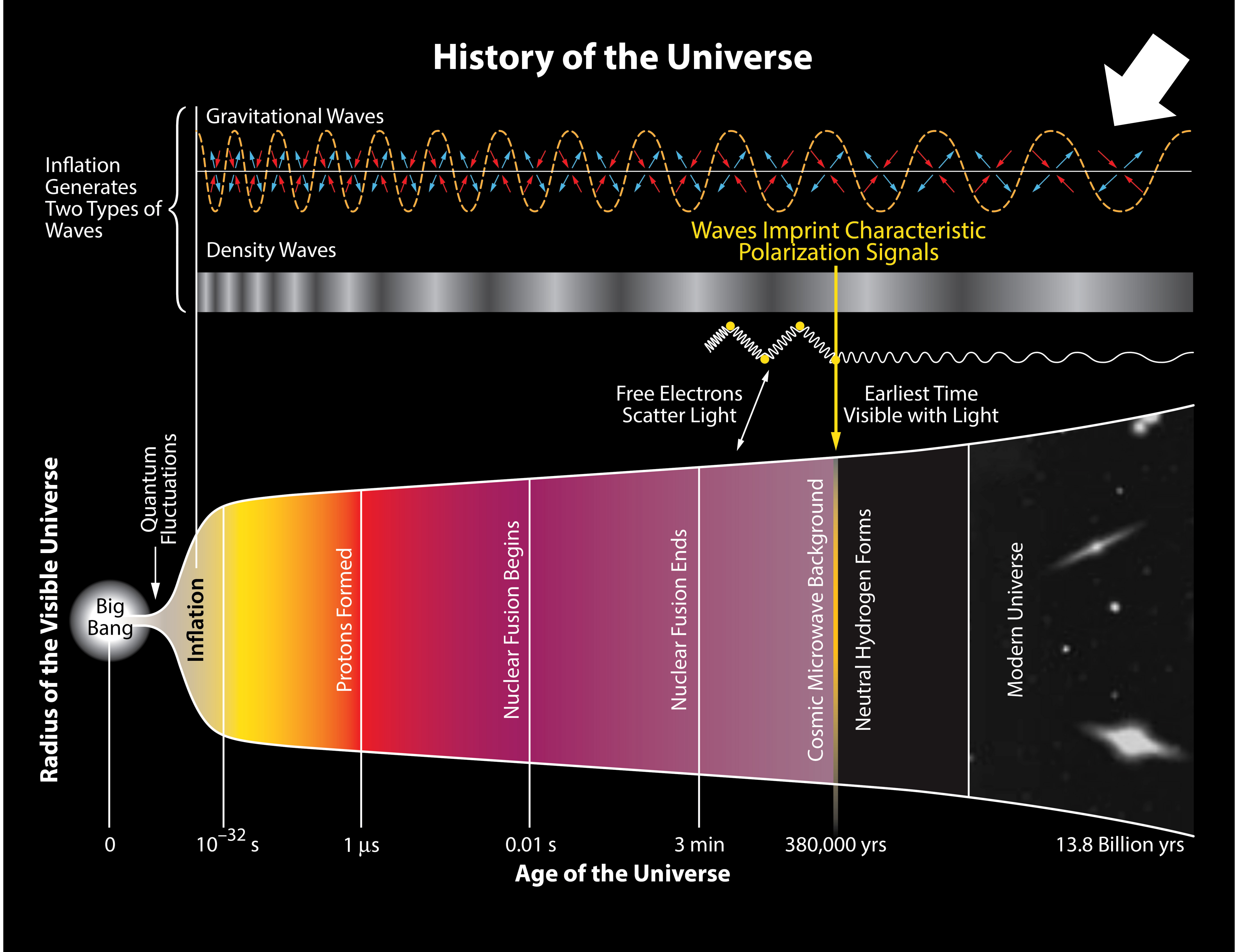
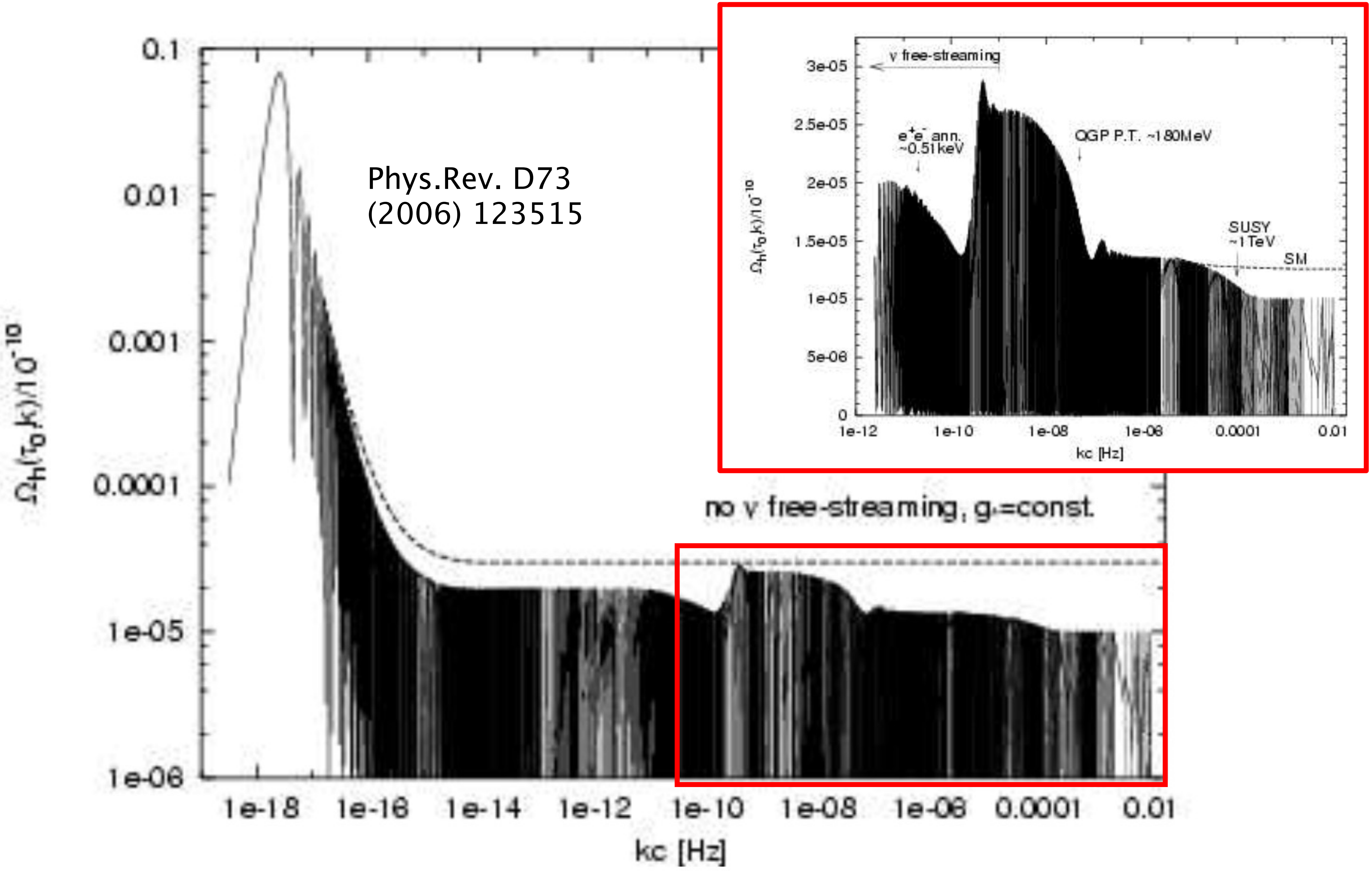

History of the Universe
Gravitational Waves
Inflation Generates Two Types of Waves
Density Waves
Waves Imprint Characteristic Polarization Signals
Free Electrons Scatter Light
Earliest Time Visible with Light
Quantum Fluctuations
Big Bang
Inflation
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Neutral Hydrogen Forms
Modern Universe
Radius of the Visible Universe
0
10^{-32} s
1 μs
0.01 s
3 min
380,000 yrs
13.8 Billion yrs
Age of the Universe

Phys.Rev. D73 (2006) 123515



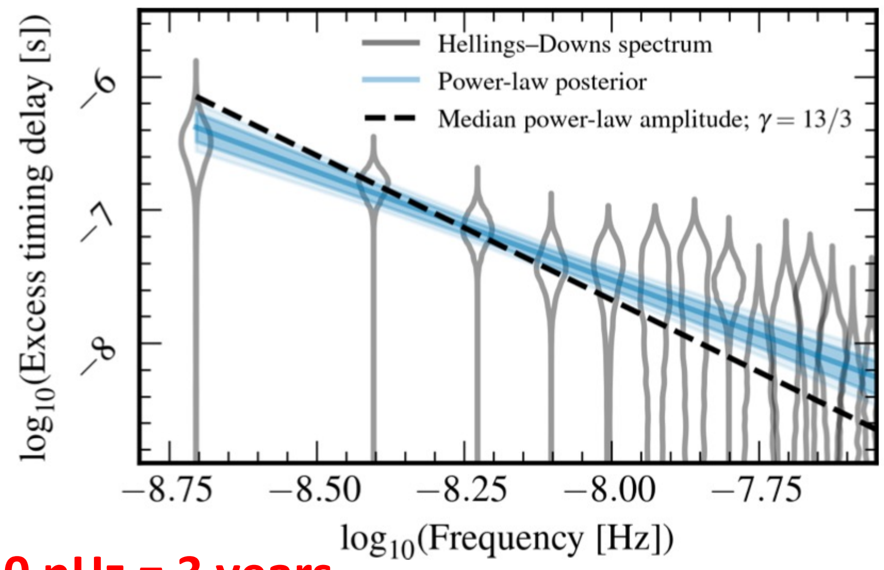
News from astronomy: pulsars (15 yr) arXiv:2306.16213v1

The NANOGrav 15-year Data Set: Evidence for a Gravitational-Wave Background

Gabriella Agazie,[1] Akash Anumarlapudi,[1] Anne M. Archibald,[2] Zaven Arzoumanian,[3] Paul T. Baker,[4] Bence Bécsy,[5] Laura Blecha,[6] Adam Brazier,[7,8] Paul R. Brook,[9] Sarah Burke-Spolaor,[10,11] Rand Burnette,[5] Robin Case,[5] Maria Charisi,[12] Shami Chatterjee,[7] Katerina Chatziioannou,[13] Belinda D. Cheeseboro,[10,11] Siyuan Chen,[14] Tyler Cohen,[15] James M. Cordes,[7] Neil J. Cornish,[16] Fronefield Crawford,[17] H. Thankful Cromartie,[7,*] Kathryn Crowter,[18] Curt J. Cutler,[19,13] Megan E. DeCesar,[20] Dallas DeGan,[5] Paul B. Demorest,[21] Heling Deng,[5] Timothy Dolch,[22,23] Brendan Drachler,[24,25] Justin A. Ellis,[26] Elizabeth C. Ferrara,[27,28,29] William Fiore,[10,11] Emmanuel Fonseca,[10,11] Gabriel E. Freedman,[1] Nate Garver-Daniels,[10,11] Peter A. Gentile,[10,11] Kyle A. Gersbach,[12] Joseph Glaser,[10,11] Deborah C. Good,[30,31] Kayhan Gültekin,[32] Jeffrey S. Hazboun,[5] Sophie Hourihane,[13] Kristina Islo,[1] Ross J. Jennings,[10,11,†] Aaron D. Johnson,[1,13] Megan L. Jones,[1] Andrew R. Kaiser,[10,11] David L. Kaplan,[1] Luke Zoltan Kelley,[33] Matthew Kerr,[34] Joey S. Key,[35] Tonia C. Klein,[1] Nima Laal,[5] Michael T. Lam,[24,25] William G. Lamb,[12] T. Joseph W. Lazio,[19] Natalia Lewandowska,[36] Tyson B. Littenberg,[37] Tingting Liu,[10,11] Andrea Lommen,[38] Duncan R. Lorimer,[10,11] Jing Luo,[39,‡] Ryan S. Lynch,[40] Chung-Pei Ma,[33,41] Dustin R. Madison,[42] Margaret A. Mattson,[10,11] Alexander McEwen,[1] James W. McKee,[43,44] Maura A. McLaughlin,[10,11] Natasha McMann,[12] Bradley W. Meyers,[18,45] Patrick M. Meyers,[13] Chiara M. F. Mingarelli,[31,30,46] Andrea Mitridate,[47] Priyamvada Natarajan,[48,49] Cherry Ng,[50] David J. Nice,[51] Stella Koch Ocker,[7] Ken D. Olum,[52] Timothy T. Pennucci,[53] Benetge B. P. Perera,[54] Polina Petrov,[12] Nihan S. Pol,[12] Henri A. Radovan,[55] Scott M. Ransom,[56] Paul S. Ray,[34] Joseph D. Romano,[57] Shashwat C. Sardesai,[1] Ann Schmiedekamp,[58] Carl Schmiedekamp,[58] Kai Schmitz,[59] Levi Schult,[12] Brent J. Shapiro-Albert,[10,11,60] Xavier Siemens,[5,1] Joseph Simon,[61,§] Magdalena S. Siwek,[62] Ingrid H. Stairs,[18] Daniel R. Stinebring,[63] Kevin Stovall,[21] Jerry P. Sun,[5] Abhimanyu Susobhanan,[1] Joseph K. Swiggum,[51,†] Jacob Taylor,[5] Stephen R. Taylor,[12] Jacob E. Turner,[10,11] Caner Unal,[64,65] Michele Vallisneri,[19,13] Rutger van Haasteren,[66] Sarah J. Vigeland,[1] Haley M. Wahl,[10,11] Qiaohong Wang,[12] Caitlin A. Witt,[67,68] Olivia Young,[24,25]

The NANOGrav Collaboration

(a)



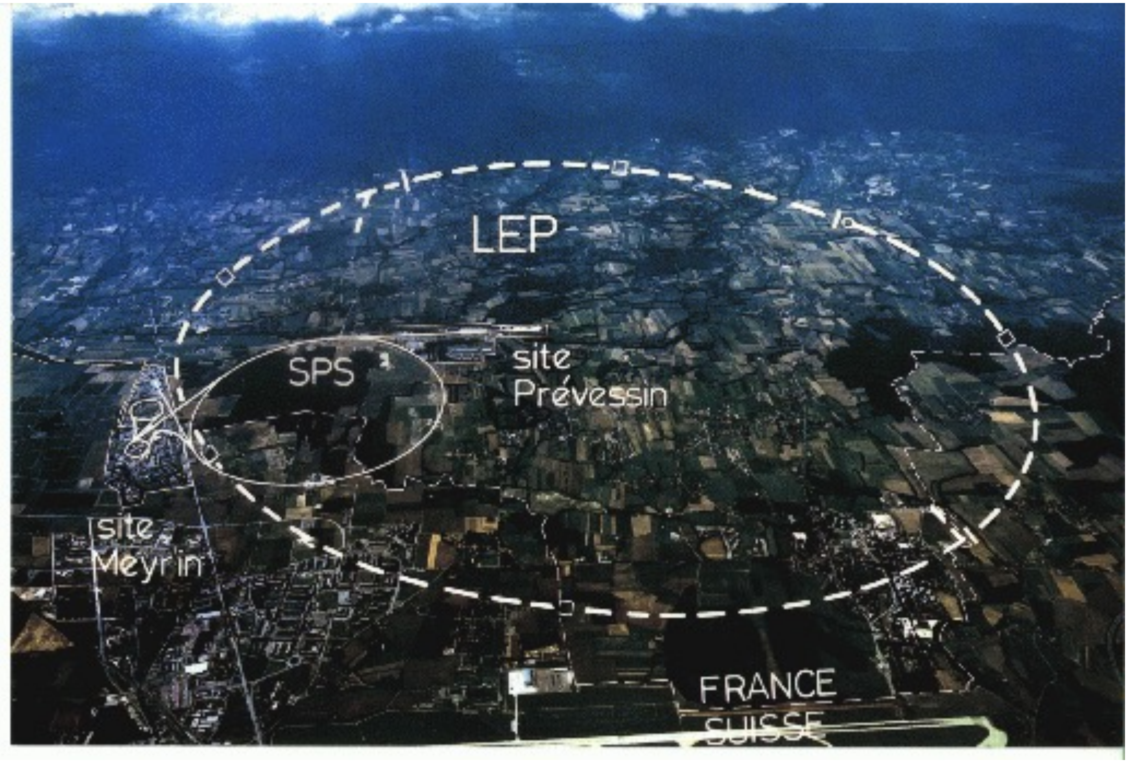
10 nHz = 3 years

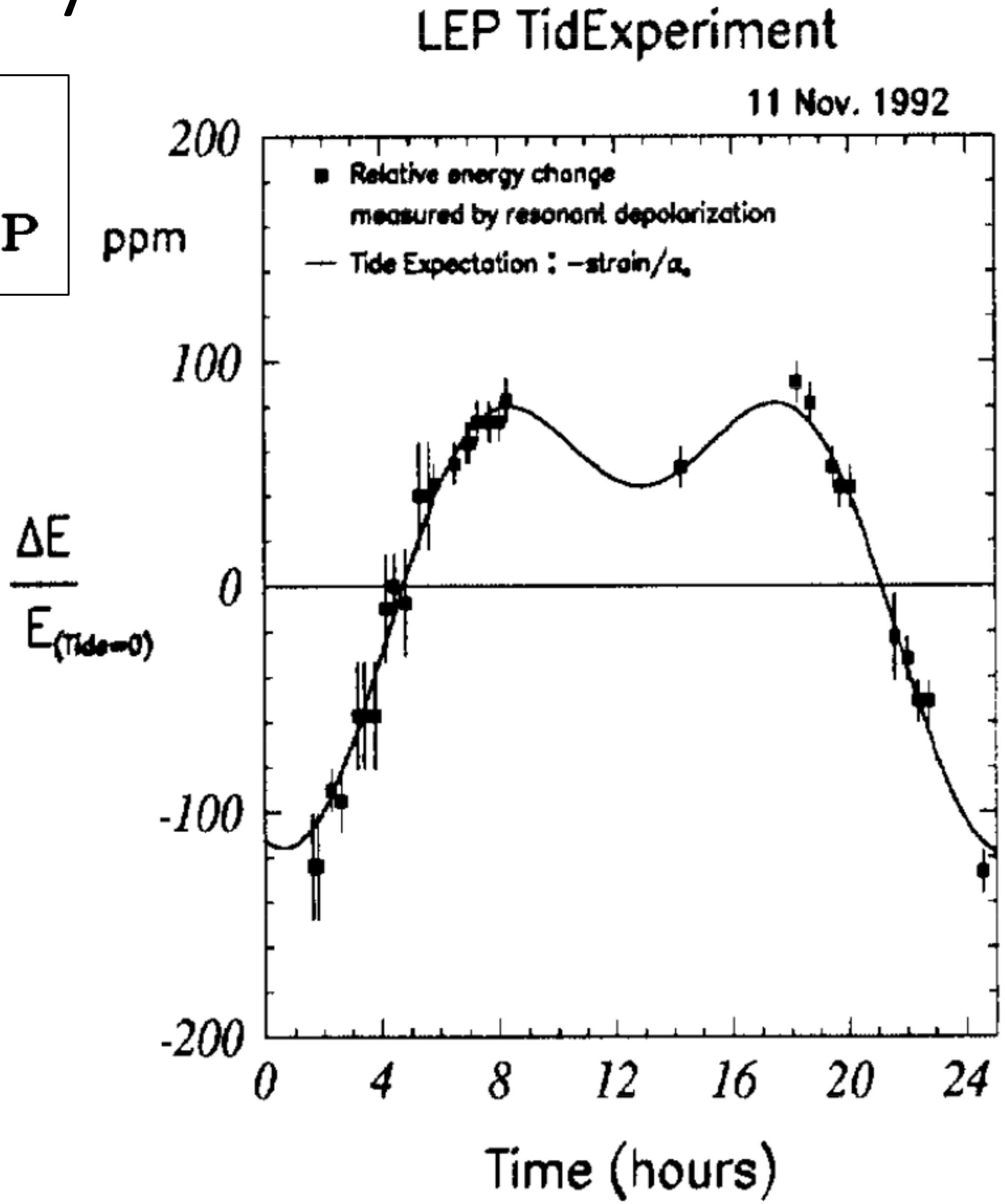
- They claim an evidence of stochastic GW background at very low frequency (**nHz**)
- Their results are consistent with massive black hole mergers but do not exclude other possibilities

However, GWB signals that are not produced by populations of inspiraling black holes may also lie within the nHz band; these include primordial GWs from inflation, scalar-induced GWs, and GW signals from multiple processes arising due to cosmological phase transitions, such as collisions of bubbles of the post-transition vacuum state, sound waves, turbulence, and the decay of any defects such as cosmic strings or domain walls that may have formed (see, e.g., Guzzetti et al. 2016; Caprini & Figueroa 2018; Domènech 2021, and references therein).

Re-discovery of the Moon by LEP

Effects of Tidal Forces on the Beam Energy in LEP





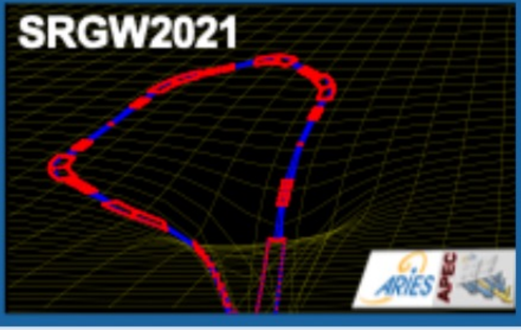
→ 24 hours $\sim 1.1 \times 10^{-5}$ Hz

Naïve insights

- Accelerators already observed gravitational tides
- But not gravitational waves
  - Gravitational waves are quadrupole
  - X and y strain may be cancelled
  - Beam trajectory may not be influenced by GW
- The sensitivity may not be enough
  - But what determines the sensitivity / resolution ?
  - How to improve? Feasibility?

→ Let us carefully investigate the GW interaction with accelerators

Theory



SRGW2021 - ARIES WP6 Workshop: Storage Rings and Gravitational Waves

2 February 2021 to 31 March 2021
Europe/Zurich timezone

Enter your search term

Storage Rings and Gravitational Waves: Summary and Outlook

A. Berlin[1], M. Brüggen[2], O. Buchmueller[3], P. Chen[4], R. T. D'Agnolo[5], R. Deng[6], J. R. Ellis[7,×,*], S. Ellis[5], G. Franchetti[8], A. Ivanov[9], J. M. Jowett[8], A. P. Kobushkin[10], S. Y. Lee[11], J. Liske[2], K. Oide[12], S. Rao[2], J. Wenninger[13], M. Wellenzohn[9], M. Zanetti[14], F. Zimmermann[13,×,†]

arXiv:2105.00992

Based on the discussion here

Storage rings as detectors for relic gravitational-wave background ?

A. N. Ivanov,[1,*] A. P. Kobushkin,[2,†] and M. Wellenzohn[1,3,‡]

[1] *Atominstitut, Technische Universität Wien, Stadionallee 2, A-1020 Wien, Austria*
[2] *Bogoliubov Institute for Theoretical Physics, 03143, Kiev and Physical and Technical National University KPI, Prospect Pobedy 37, 03056, Kiev, Ukraine*
[3] *FH Campus Wien, University of Applied Sciences, Favoritenstraße 226, 1100 Wien, Austria*
(Dated: March 31, 2021)

arXiv:gr-qc/0210091

Conventional detection of GW

The Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

To the linear order with small h

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

In vacuum $T_{\mu\nu} = 0$

$$\Gamma^{\mu}{}_{\alpha\beta} = g^{\mu\nu}\Gamma_{\nu\alpha\beta} = \frac{1}{2}\left(h^{\mu}{}_{\alpha,\beta} + h^{\mu}{}_{\beta,\alpha} - h_{\alpha\beta}{}^{,\mu}\right) + O(h^2)$$

$$R^{\alpha}{}_{\beta\gamma\delta} = \frac{1}{2}\left(h^{\alpha}{}_{\delta,\beta\gamma} - h_{\beta\delta}{}^{,\alpha}{}_{,\gamma} - h^{\alpha}{}_{\gamma,\beta\delta} + h_{\beta\gamma}{}^{,\alpha}{}_{,\delta}\right) + O(h^2)$$

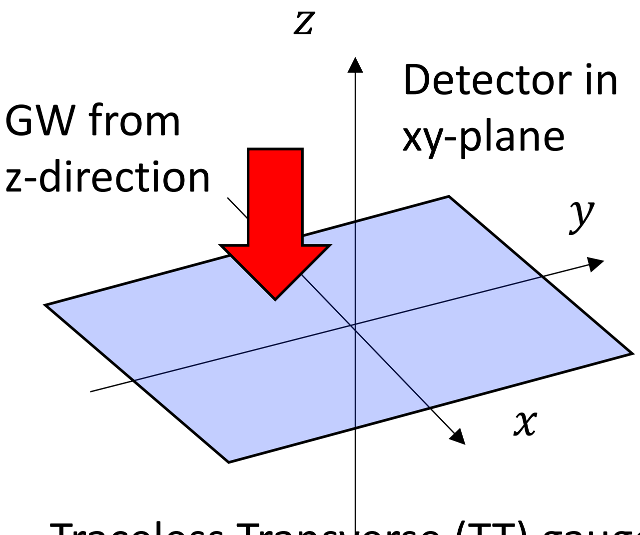
Propagation of GW

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) h_{\mu\nu} = 0$$

Relevant elements of the Riemann tensor

$$R^{x}{}_{0x0} = -R^{y}{}_{0y0} = +\frac{1}{2}\frac{d^2 h_{xx}}{dt^2}$$

$$R^{y}{}_{0y0} = R^{y}{}_{0x0} = -\frac{1}{2}\frac{d^2 h_{xx}}{dt^2}$$



Traceless Transverse (TT) gauge

$$h_{xx} = -h_{yy} = \Delta_{+} \cos(\omega t - kz + \delta)$$

$$h_{xy} = h_{yx} = \Delta_{\times} \cos(\omega t - kz + \delta)$$

→

Detection of GW

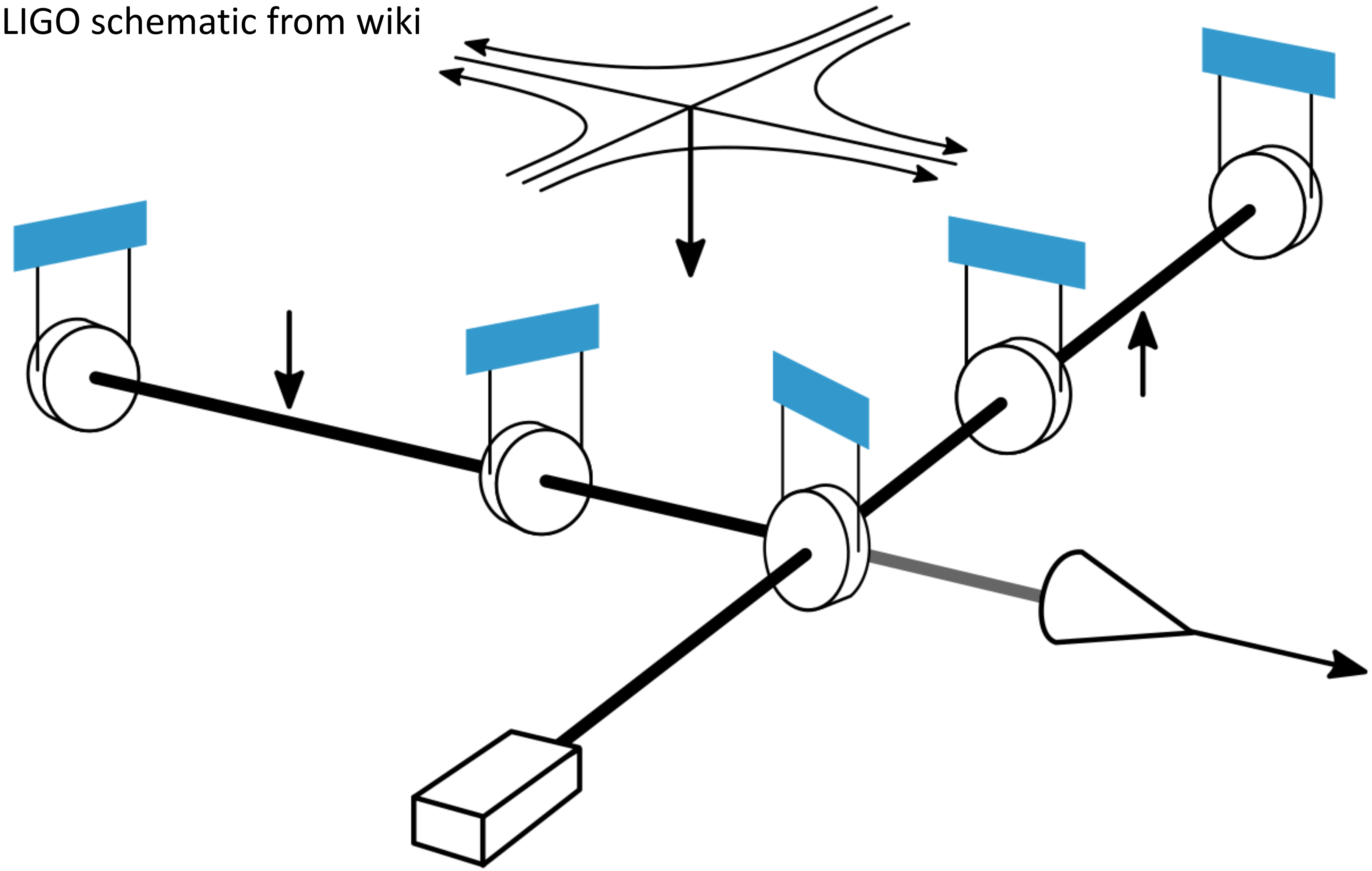
$$\frac{d^2 x}{dt^2} = -R_{x0x0}\, x - R_{x0y0}\, y$$

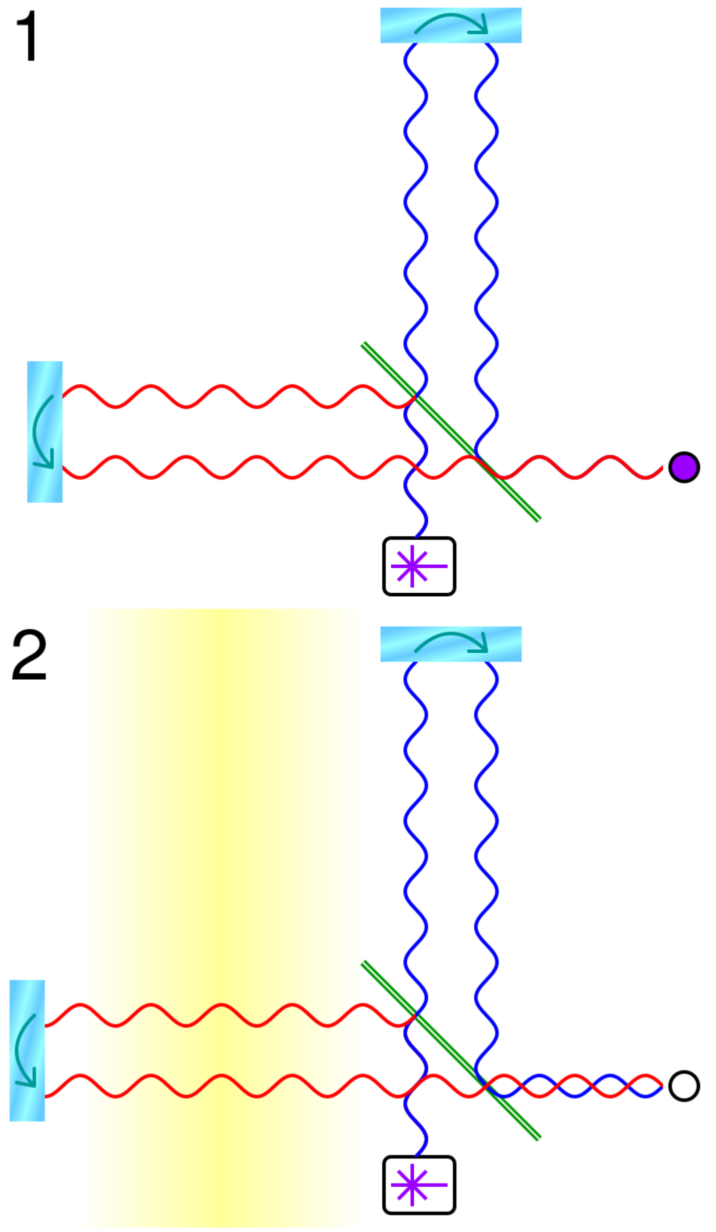
$$\frac{d^2 y}{dt^2} = -R_{y0x0}\, x - R_{y0y0}\, y$$

Structure oscillates in x and y

GW Laser interferometer

LIGO schematic from wiki





How about the 2nd order in $h_{\mu\nu}$?

$$\Gamma^{\mu}{}_{\alpha\beta} = \frac{1}{2}\eta^{\mu\nu}\left(h_{\nu\alpha,\beta} + h_{\nu\beta,\alpha} - h_{\alpha\beta,\nu}\right) - \frac{1}{2}h^{\mu\nu}\left(h_{\nu\alpha,\beta} + h_{\nu\beta,\alpha} - h_{\alpha\beta,\nu}\right) + O(h^3)$$

40 (4x10) Christoffel symbols need to be checked

$$\Gamma^{x}{}_{0x} = \Gamma^{x}{}_{x0} = \frac{1}{2}\eta^{x\nu}\left(h_{\nu 0,x} + h_{\nu x,0} - h_{0x,\nu}\right) - \frac{1}{2}h^{x\nu}\left(h_{\nu 0,x} + h_{\nu x,0} - h_{0x,\nu}\right)$$

$$= \frac{1}{2}\left(\eta^{x0}h_{0x,0} + \eta^{xx}h_{xx,0} + \eta^{xy}h_{yx,0} + \eta^{xz}h_{zx,0}\right) - \frac{1}{2}\left(h^{x0}h_{0x,0} + h^{xx}h_{xx,0} + h^{xy}h_{yx,0} + h^{xz}h_{zx,0}\right)$$

$$= \frac{1}{2}\left(h_{xx,0} - h^{xx}h_{xx,0} - h^{xy}h_{yx,0}\right) = \frac{1}{2}\frac{\partial h_{xx}}{\partial t} - \frac{1}{2}h^{xx}\frac{\partial h_{xx}}{\partial t} - \frac{1}{2}h^{xy}\frac{\partial h_{yx}}{\partial t}$$

$$\Gamma^{x}{}_{0y} = \Gamma^{x}{}_{y0} = \frac{1}{2}\eta^{x\nu}\left(h_{\nu 0,y} + h_{\nu y,0} - h_{0y,\nu}\right) - \frac{1}{2}h^{x\nu}\left(h_{\nu 0,y} + h_{\nu y,0} - h_{0y,\nu}\right)$$

$$= \frac{1}{2}\left(\eta^{x0}h_{0y,0} + \eta^{xx}h_{xy,0} + \eta^{xy}h_{yy,0} + \eta^{xz}h_{zy,0}\right) - \frac{1}{2}\left(h^{x0}h_{0y,0} + h^{xx}h_{xy,0} + h^{xy}h_{yy,0} + h^{xz}h_{zy,0}\right)$$

$$= \frac{1}{2}\left(h_{xy,0} - h^{xx}h_{xy,0} - h^{xy}h_{yy,0}\right) = \frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xx}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xy}\frac{\partial h_{yy}}{\partial t}$$

…continue for other 36 terms

Summary of non-zero $\Gamma^{\lambda}{}_{\mu\nu}$ under TT gauge

$$\Gamma^{x}{}_{0x} = \Gamma^{x}{}_{x0} = \frac{1}{2}\frac{\partial h_{xx}}{\partial t} - \frac{1}{2}h^{xx}\frac{\partial h_{xx}}{\partial t} - \frac{1}{2}h^{xy}\frac{\partial h_{yx}}{\partial t}$$

$$\Gamma^{x}{}_{0y} = \Gamma^{x}{}_{y0} = \frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xx}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xy}\frac{\partial h_{yy}}{\partial t}$$

$$\Gamma^{y}{}_{0y} = \Gamma^{y}{}_{y0} = \frac{1}{2}\frac{\partial h_{yy}}{\partial t} - \frac{1}{2}h^{yy}\frac{\partial h_{yy}}{\partial t} - \frac{1}{2}h^{yx}\frac{\partial h_{xy}}{\partial t}$$

$$\Gamma^{y}{}_{0x} = \Gamma^{y}{}_{x0} = \frac{1}{2}\frac{\partial h_{yx}}{\partial t} - \frac{1}{2}h^{yy}\frac{\partial h_{yx}}{\partial t} - \frac{1}{2}h^{yx}\frac{\partial h_{xx}}{\partial t}$$

They contribute to the signal

$$\Gamma^{0}{}_{xx} = -\frac{1}{2}\frac{\partial h_{xx}}{\partial t}$$

$$\Gamma^{0}{}_{xy} = \Gamma^{0}{}_{yx} = \frac{1}{2}\frac{\partial h_{xy}}{\partial t}$$

$$\Gamma^{0}{}_{yy} = -\frac{1}{2}\frac{\partial h_{yy}}{\partial t}$$

$$\Gamma^{x}{}_{xz} = \Gamma^{x}{}_{zx} = -\frac{1}{2}\frac{\partial h_{xx}}{\partial z} - \frac{1}{2}h^{xx}\frac{\partial h_{xx}}{\partial z} - \frac{1}{2}h^{xy}\frac{\partial h_{yx}}{\partial z}$$

$$\Gamma^{x}{}_{yz} = \Gamma^{x}{}_{zy} = -\frac{1}{2}\frac{\partial h_{xy}}{\partial z} - \frac{1}{2}h^{xx}\frac{\partial h_{xy}}{\partial z} - \frac{1}{2}h^{xy}\frac{\partial h_{yy}}{\partial z}$$

$$\Gamma^{y}{}_{yz} = \Gamma^{y}{}_{zy} = -\frac{1}{2}\frac{\partial h_{yy}}{\partial z} - \frac{1}{2}h^{yy}\frac{\partial h_{yy}}{\partial z} - \frac{1}{2}h^{yx}\frac{\partial h_{xy}}{\partial z}$$

$$\Gamma^{y}{}_{xz} = \Gamma^{y}{}_{zx} = -\frac{1}{2}\frac{\partial h_{yx}}{\partial z} - \frac{1}{2}h^{yy}\frac{\partial h_{yx}}{\partial z} - \frac{1}{2}h^{yx}\frac{\partial h_{xx}}{\partial z}$$

$$\Gamma^{z}{}_{xx} = -\frac{1}{2}\frac{\partial h_{xx}}{\partial z}$$

$$\Gamma^{z}{}_{xy} = \Gamma^{z}{}_{yx} = \frac{1}{2}\frac{\partial h_{xy}}{\partial z}$$

$$\Gamma^{z}{}_{yy} = -\frac{1}{2}\frac{\partial h_{yy}}{\partial z}$$

14 out of 40 components remain

Simplification of $\Gamma^{\lambda}{}_{\mu\nu}$ under TT gauge

Condition: $h_{xx} = -h_{yy},\ \ h_{xy} = h_{yx}$

$$\Gamma^{x}{}_{0x} = \Gamma^{x}{}_{x0} = \frac{1}{2}\frac{\partial h_{xx}}{\partial t} - \frac{1}{2}h^{xx}\frac{\partial h_{xx}}{\partial t} - \frac{1}{2}h^{xy}\frac{\partial h_{yx}}{\partial t} = +\frac{1}{2}\frac{\partial h_{xx}}{\partial t} - \frac{1}{4}\frac{\partial}{\partial t}\left[h_{xx}^2 + h_{xy}^2\right]$$

$$\Gamma^{y}{}_{0y} = \Gamma^{y}{}_{y0} = \frac{1}{2}\frac{\partial h_{yy}}{\partial t} - \frac{1}{2}h^{yy}\frac{\partial h_{yy}}{\partial t} - \frac{1}{2}h^{yx}\frac{\partial h_{xy}}{\partial t} = -\frac{1}{2}\frac{\partial h_{xx}}{\partial t} - \frac{1}{4}\frac{\partial}{\partial t}\left[h_{xx}^2 + h_{xy}^2\right]$$

$$\Gamma^{x}{}_{0y} = \Gamma^{x}{}_{y0} = \frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xx}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xy}\frac{\partial h_{yy}}{\partial t} = \frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xx}\frac{\partial h_{xy}}{\partial t} + \frac{1}{2}h^{xy}\frac{\partial h_{xx}}{\partial t}$$

$$= \frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{h_{xx}h_{xy}}{2}\left(\frac{h_{xx}}{h_{xy}}\right)\frac{\left(\partial h_{xy}/\partial t\right)h_{xx} - \left(\partial h_{xx}/\partial t\right)h_{xy}}{h_{xx}^2} = \frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{h_{xx}h_{xy}}{2}\frac{\partial}{\partial t}\ln\left(\frac{h_{xy}}{h_{xx}}\right)$$

$$\Gamma^{y}{}_{0x} = \Gamma^{y}{}_{x0} = \frac{1}{2}\frac{\partial h_{yx}}{\partial t} - \frac{1}{2}h^{yy}\frac{\partial h_{yx}}{\partial t} - \frac{1}{2}h^{yx}\frac{\partial h_{xx}}{\partial t} = \frac{1}{2}\frac{\partial h_{xy}}{\partial t} + \frac{1}{2}h^{xx}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xy}\frac{\partial h_{xx}}{\partial t}$$

$$= \frac{1}{2}\frac{\partial h_{xy}}{\partial t} + \frac{h_{xx}h_{xy}}{2}\left(\frac{h_{xx}}{h_{xy}}\right)\frac{\left(\partial h_{xy}/\partial t\right)h_{xx} - \left(\partial h_{xx}/\partial t\right)h_{xy}}{h_{xx}^2} = \frac{1}{2}\frac{\partial h_{xy}}{\partial t} + \frac{h_{xx}h_{xy}}{2}\frac{\partial}{\partial t}\ln\left(\frac{h_{xy}}{h_{xx}}\right)$$

Riemann tensor under the gauge condition

$$R^x{}_{0x0} = \Gamma^x{}_{00,x} - \Gamma^x{}_{0x,0} + \Gamma^x{}_{\mu x}\Gamma^\mu{}_{00} - \Gamma^x{}_{\mu 0}\Gamma^\mu{}_{0x}$$

$$\Gamma^x{}_{0x,0} = \frac{\partial}{\partial t}\left[\frac{1}{2}\frac{\partial h_{xx}}{\partial t} - \frac{1}{4}\frac{\partial}{\partial t}\left[h_{xx}^2 + h_{xy}^2\right]\right] = \frac{1}{2}\frac{\partial^2 h_{xx}}{\partial t^2} - \frac{1}{4}\frac{\partial^2}{\partial t^2}\left[h_{xx}^2 + h_{xy}^2\right]$$

$$\Gamma^x{}_{\mu 0}\Gamma^\mu{}_{0x} = \Gamma^x{}_{00}\Gamma^0{}_{0x} + \Gamma^x{}_{x0}\Gamma^x{}_{0x} + \Gamma^x{}_{y0}\Gamma^y{}_{0x} + \Gamma^x{}_{z0}\Gamma^z{}_{0x}$$

$$= \left[\frac{1}{2}\frac{\partial h_{xx}}{\partial t} - \frac{1}{4}\frac{\partial}{\partial t}\left[h_{xx}^2 + h_{xy}^2\right]\right]^2 + \left[\frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{h_{xx}h_{xy}}{2}\frac{\partial}{\partial t}\ln\left(\frac{h_{xy}}{h_{xx}}\right)\right]\left[\frac{1}{2}\frac{\partial h_{xy}}{\partial t} + \frac{h_{xx}h_{xy}}{2}\frac{\partial}{\partial t}\ln\left(\frac{h_{xy}}{h_{xx}}\right)\right]$$

$$= \frac{1}{4}\left(\frac{\partial h_{xx}}{\partial t}\right)^2 + \frac{1}{4}\left(\frac{\partial h_{xy}}{\partial t}\right)^2 + O(h^3)$$

$$\rightarrow R^x{}_{0x0} = \frac{1}{2}\frac{\partial^2 h_{xx}}{\partial t^2} - \frac{1}{4}\frac{\partial^2}{\partial t^2}\left[h_{xx}^2 + h_{xy}^2\right] + \frac{1}{4}\left[\left(\frac{\partial h_{xx}}{\partial t}\right)^2 + \left(\frac{\partial h_{xy}}{\partial t}\right)^2\right] + O(h^3)$$

Further calculations

$$R^{x}{}_{0y0} = \Gamma^{x}{}_{00,y} - \Gamma^{x}{}_{0y,0} + \Gamma^{x}{}_{\mu y}\Gamma^{\mu}{}_{00} - \Gamma^{x}{}_{\mu 0}\Gamma^{\mu}{}_{0y}$$

$$\Gamma^{x}{}_{0y,0} = \frac{\partial}{\partial t}\left[\frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xx}\frac{\partial h_{xy}}{\partial t} + \frac{1}{2}h^{xy}\frac{\partial h_{xx}}{\partial t}\right] = \frac{1}{2}\frac{\partial^2 h_{xy}}{\partial t^2} - \frac{1}{2}\frac{\partial h_{xx}}{\partial t}\frac{\partial h_{xy}}{\partial t} - \frac{1}{2}h^{xx}\frac{\partial^2 h_{xy}}{\partial t^2} + \frac{1}{2}\frac{\partial h_{xy}}{\partial t}\frac{\partial h_{xx}}{\partial t} + \frac{1}{2}h^{xy}\frac{\partial^2 h_{xx}}{\partial t^2}$$

$$\Gamma^{x}{}_{\mu 0}\Gamma^{\mu}{}_{0y} = \Gamma^{x}{}_{00}\Gamma^{0}{}_{0y} + \Gamma^{x}{}_{x0}\Gamma^{x}{}_{0y} + \Gamma^{x}{}_{y0}\Gamma^{y}{}_{0y} + \Gamma^{x}{}_{z0}\Gamma^{z}{}_{0y}$$

$$= \left\{\frac{1}{2}\frac{\partial h_{xx}}{\partial t} - \frac{1}{4}\frac{\partial}{\partial t}\left[h_{xx}^2 + h_{xy}^2\right]\right\}\left\{\frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{h_{xx}h_{xy}}{2}\frac{\partial}{\partial t}\ln\left(\frac{h_{xy}}{h_{xx}}\right)\right\} + \left\{\frac{1}{2}\frac{\partial h_{xy}}{\partial t} - \frac{h_{xx}h_{xy}}{2}\frac{\partial}{\partial t}\ln\left(\frac{h_{xy}}{h_{xx}}\right)\right\}\left\{-\frac{1}{2}\frac{\partial h_{xx}}{\partial t} - \frac{1}{4}\frac{\partial}{\partial t}\left[h_{xx}^2 + h_{xy}^2\right]\right\}$$

$$= \frac{1}{4}\left(\frac{\partial h_{xx}}{\partial t}\right)\left(\frac{\partial h_{xy}}{\partial t}\right) - \frac{1}{4}\left(\frac{\partial h_{xx}}{\partial t}\right)\left(\frac{\partial h_{xy}}{\partial t}\right) + O(h^3)$$

Not in arXiv:gr-qc/0210091

$$\rightarrow R^{x}{}_{0y0} = \frac{1}{2}\frac{\partial^2 h_{xy}}{\partial t^2} \boxed{- \frac{1}{2}h^{xx}\frac{\partial^2 h_{xy}}{\partial t^2} + \frac{1}{2}h^{xy}\frac{\partial^2 h_{xx}}{\partial t^2}} + O(h^3)$$

Combine all

Not in arXiv:gr-qc/0210091

$$\begin{cases} \dfrac{d^2x}{dt^2} = -\left\{\dfrac{1}{2}\dfrac{\partial^2 h_{xx}}{\partial t^2} - \dfrac{1}{4}\dfrac{\partial^2}{\partial t^2}\left[h_{xx}^2 + h_{xy}^2\right] + \dfrac{1}{4}\left[\left(\dfrac{\partial h_{xx}}{\partial t}\right)^2 + \left(\dfrac{\partial h_{xy}}{\partial t}\right)^2\right]\right\}x - \left\{\dfrac{1}{2}\dfrac{\partial^2 h_{xy}}{\partial t^2} \boxed{-\dfrac{1}{2}h^{xx}\dfrac{\partial^2 h_{xy}}{\partial t^2} + \dfrac{1}{2}h^{xy}\dfrac{\partial^2 h_{xx}}{\partial t^2}}\right\}y \\ \dfrac{d^2y}{dt^2} = -\left\{\dfrac{1}{2}\dfrac{\partial^2 h_{xy}}{\partial t^2} \boxed{+\dfrac{1}{2}h^{xx}\dfrac{\partial^2 h_{xy}}{\partial t^2} - \dfrac{1}{2}h^{xy}\dfrac{\partial^2 h_{xx}}{\partial t^2}}\right\}x - \left\{-\dfrac{1}{2}\dfrac{\partial^2 h_{xx}}{\partial t^2} - \dfrac{1}{4}\dfrac{\partial^2}{\partial t^2}\left[h_{xx}^2 + h_{xy}^2\right] + \dfrac{1}{4}\left[\left(\dfrac{\partial h_{xx}}{\partial t}\right)^2 + \left(\dfrac{\partial h_{xy}}{\partial t}\right)^2\right]\right\}y \end{cases}$$

Not in arXiv:gr-qc/0210091

Cylindrical coordinate $x = r\cos\Phi \quad y = r\sin\Phi$

$$\frac{dx}{dt} = \frac{dr}{dt}\cos\Phi - \frac{rd\Phi}{dt}\sin\Phi \quad \rightarrow \frac{d^2x}{dt^2} = \frac{d^2r}{dt^2}\cos\Phi - 2\frac{dr}{dt}\frac{d\Phi}{dt}\sin\Phi - \frac{rd^2\Phi}{dt^2}\sin\Phi - r\left(\frac{d\Phi}{dt}\right)^2\cos\Phi$$

$$\frac{dy}{dt} = \frac{dr}{dt}\sin\Phi + \frac{rd\Phi}{dt}\cos\Phi \quad \rightarrow \frac{d^2y}{dt^2} = \frac{d^2r}{dt^2}\sin\Phi + 2\frac{dr}{dt}\frac{d\Phi}{dt}\cos\Phi + \frac{rd^2\Phi}{dt^2}\cos\Phi - r\left(\frac{d\Phi}{dt}\right)^2\sin\Phi$$

$$\cos\Phi\frac{d^2x}{dt^2} + \sin\Phi\frac{d^2y}{dt^2} = \frac{d^2r}{dt^2} - r\left(\frac{d\Phi}{dt}\right)^2$$

$$-\sin\Phi\frac{d^2x}{dt^2} + \cos\Phi\frac{d^2y}{dt^2} = 2\frac{dr}{dt}\frac{d\Phi}{dt} + \frac{rd^2\Phi}{dt^2} = \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\Phi}{dt}\right)$$

$$\begin{cases} x\dfrac{d^2x}{dt^2} + y\dfrac{d^2y}{dt^2} = r\dfrac{d^2r}{dt^2} - r^2\left(\dfrac{d\Phi}{dt}\right)^2 \\ -y\dfrac{d^2x}{dt^2} + x\dfrac{d^2y}{dt^2} = \dfrac{d}{dt}\left(r^2\dfrac{d\Phi}{dt}\right) \end{cases}$$

Further calculations

$$r\frac{d^2r}{dt^2}-r^2\left(\frac{d\Phi}{dt}\right)^2=x\frac{d^2x}{dt^2}+y\frac{d^2y}{dt^2}$$

1st order term cancels out

$$=-\left\{\frac{1}{2}\frac{\partial^2 h_{xx}}{\partial t^2}-\frac{1}{4}\frac{\partial^2}{\partial t^2}[h_{xx}^2+h_{xy}^2]+\frac{1}{4}\left[\left(\frac{\partial h_{xx}}{\partial t}\right)^2+\left(\frac{\partial h_{xy}}{\partial t}\right)^2\right]\right\}x^2-\left\{\frac{1}{2}\frac{\partial^2 h_{xy}}{\partial t^2}-\frac{1}{2}h^{xx}\frac{\partial^2 h_{xy}}{\partial t^2}+\frac{1}{2}h^{xy}\frac{\partial^2 h_{xx}}{\partial t^2}\right\}xy$$

$$-\left\{\frac{1}{2}\frac{\partial^2 h_{xy}}{\partial t^2}+\frac{1}{2}h^{xx}\frac{\partial^2 h_{xy}}{\partial t^2}-\frac{1}{2}h^{xy}\frac{\partial^2 h_{xx}}{\partial t^2}\right\}xy-\left\{-\frac{1}{2}\frac{\partial^2 h_{xx}}{\partial t^2}-\frac{1}{4}\frac{\partial^2}{\partial t^2}[h_{xx}^2+h_{xy}^2]+\frac{1}{4}\left[\left(\frac{\partial h_{xx}}{\partial t}\right)^2+\left(\frac{\partial h_{xy}}{\partial t}\right)^2\right]\right\}y^2$$

Additional term (Not in arXiv:gr-qc/0210091) cancels out

$$=\left\{\frac{1}{4}\frac{\partial^2}{\partial t^2}[h_{xx}^2+h_{xy}^2]-\frac{1}{4}\left[\left(\frac{\partial h_{xx}}{\partial t}\right)^2+\left(\frac{\partial h_{xy}}{\partial t}\right)^2\right]\right\}r^2-\frac{\partial^2 h_{xy}}{\partial t^2}xy$$

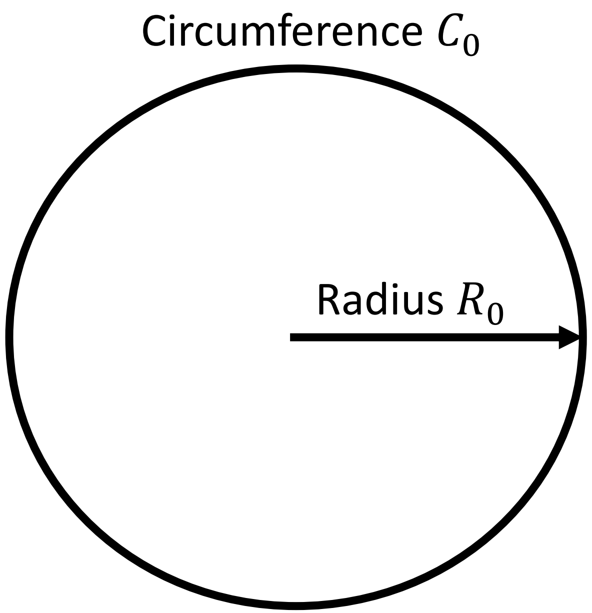
$= 0$ if change in r is small

Not in arXiv:gr-qc/0210091

$$\rightarrow\frac{1}{r}\frac{d^2r}{dt^2}=\left\{\frac{1}{4}\frac{\partial^2}{\partial t^2}[h_{xx}^2+h_{xy}^2]-\frac{1}{4}\left[\left(\frac{\partial h_{xx}}{\partial t}\right)^2+\left(\frac{\partial h_{xy}}{\partial t}\right)^2\right]\right\}-\frac{\partial^2 h_{xy}}{\partial t^2}\frac{\sin 2\Phi}{2}+\left(\frac{d\Phi}{dt}\right)^2$$

$\equiv \dot{h}^2$

The 2nd order term oscillates circular objects



The ring circumference would change by

arXiv:gr-qc/0210091

$$\frac{\Delta C}{\Delta t} = \left(\frac{\Delta C}{\Delta t}\right)_m + \left(\frac{\Delta C}{\Delta t}\right)_s + \left(\frac{\Delta C}{\Delta t}\right)_{\text{GW}}$$

tidal seasonal GW

$$\frac{1}{r}\frac{d^2 r}{dt^2} = -\frac{1}{4}\left[\left(\frac{\partial h_{xx}}{\partial t}\right)^2 + \left(\frac{\partial h_{xy}}{\partial t}\right)^2\right]$$

$$h_{xx} = \Delta_+ \cos(\omega t - kz + \delta)$$
$$h_{xy} = \Delta_\times \cos(\omega t - kz + \delta)$$
$$h_0 = \sqrt{\Delta_+^2 + \Delta_\times^2}$$

$$\frac{1}{C_0}\frac{dC(t)}{dt} = \frac{1}{R_0}\frac{dR(t)}{dt} \sim \frac{1}{16} h_0^2 \omega \sin(2\omega t + 2\delta)$$

This looks coherent but stochastic nature of relic GW may smear this signal

Coherent vs stochastic relic GW (arXiv:gr-qc/0210091)

Stochastic GW

(Rayleigh fading?)

$$\frac{\delta C_{gw}}{C_0} = -\frac{1}{16}\int_0^{\infty} d\Omega_s \int_0^{\infty} d\phi \left\langle \left(h_{xx}(t,\theta_s,\phi_s,\omega)\cos 2\phi + h_{xy}(t,\theta_s,\phi_s,\omega)\sin 2\phi\right)^2 \right\rangle_{\omega}$$

Isotopically coming GW from all solid angle

phase average

Frequency average with spectral density $S_h(\omega)$

$$\langle f \rangle_{\omega} = \int_0^{\infty} d\omega S_h(\omega) f(\omega)$$

$$\rightarrow \frac{1}{C_0}\frac{\Delta C_{gw}}{\Delta t} = \frac{4\pi}{3}\frac{1}{16} h_0^2 \int_0^{\infty} \omega S_h(\omega) \sin(2\omega t + 2\delta)\, d\omega$$

$S_h(\omega)$: spectral density

Coherent GW

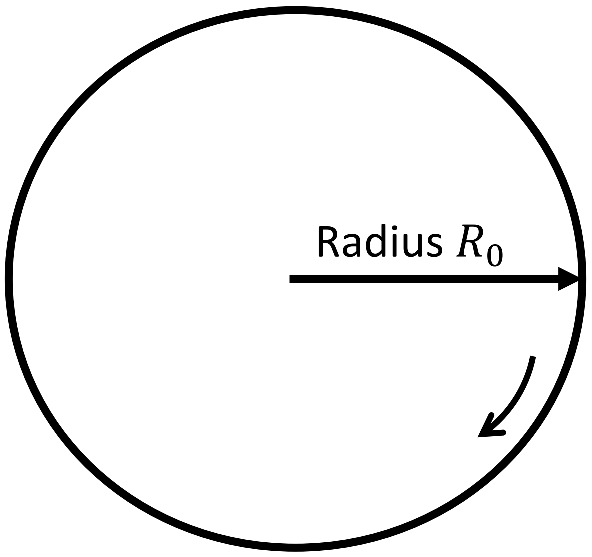
$$\frac{1}{C_0}\frac{\Delta C_{gw}}{\Delta t} = \frac{1}{16} h_0^2 \sin(2\omega t + 2\delta)$$

No major impact (unless $S(\omega)$ is uniform?)

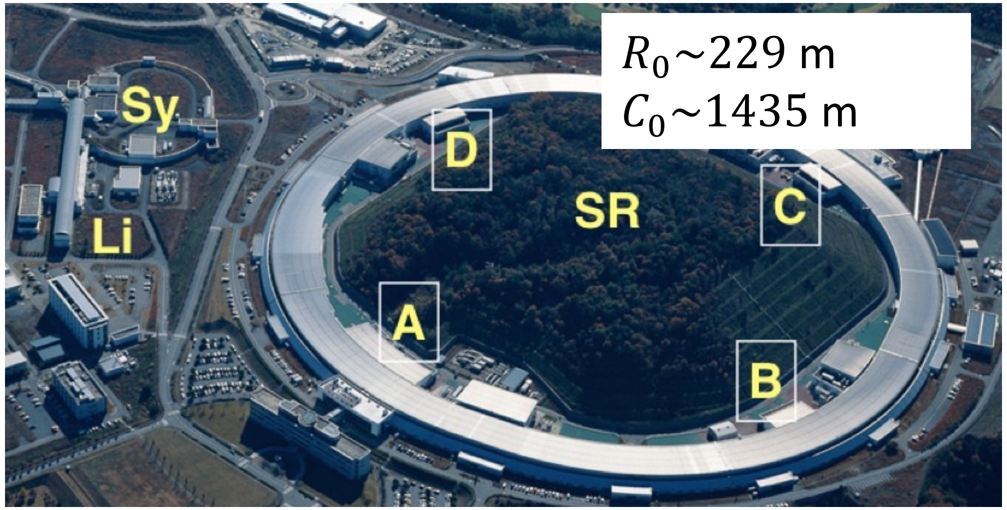
Interaction between machine circumference and GW is 2nd order

Experiment and time domain analysis

RF frequency and circumference of Spring-8



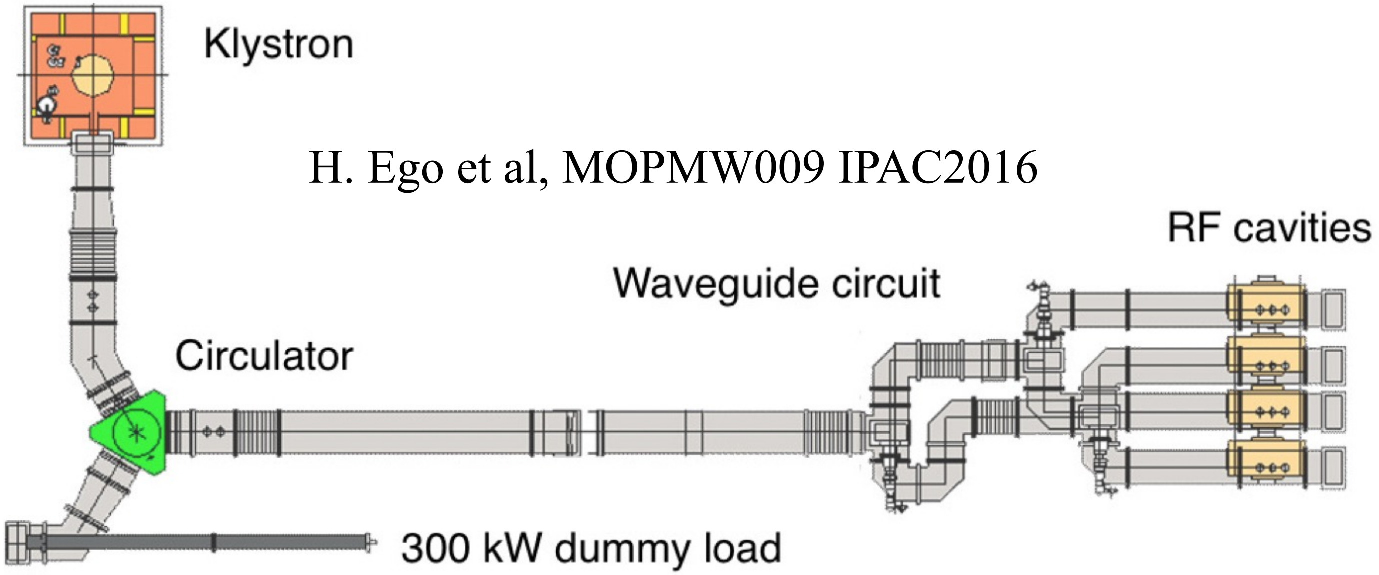
- 1 turn: T_0
- Speed of electrons v
- Beam energy 8 GeV
- $v \sim c$
- Harmonic number $n = 2436$
- RF frequency $f_{RF} = 506.756$ MHz



ABCD: RF stations

$$C_0 \sim T_0 v = \frac{c}{f_{RF}/n}$$

$C_0 = 1435.4512$ m for the official specification



H. Ego et al, MOPMW009 IPAC2016

Resolution: momentum compaction factor

$$-\frac{\Delta L}{L} \sim \frac{\Delta f}{f} = \eta_c \frac{\Delta p}{p}$$

$$\eta_c = \frac{1}{\gamma^2} - \alpha_c$$

<u>SPring-8</u>

8 GeV → $1/\gamma^2 \sim 4\times10^{-9}$

$\alpha_c \sim 1.46\times10^{-4}$

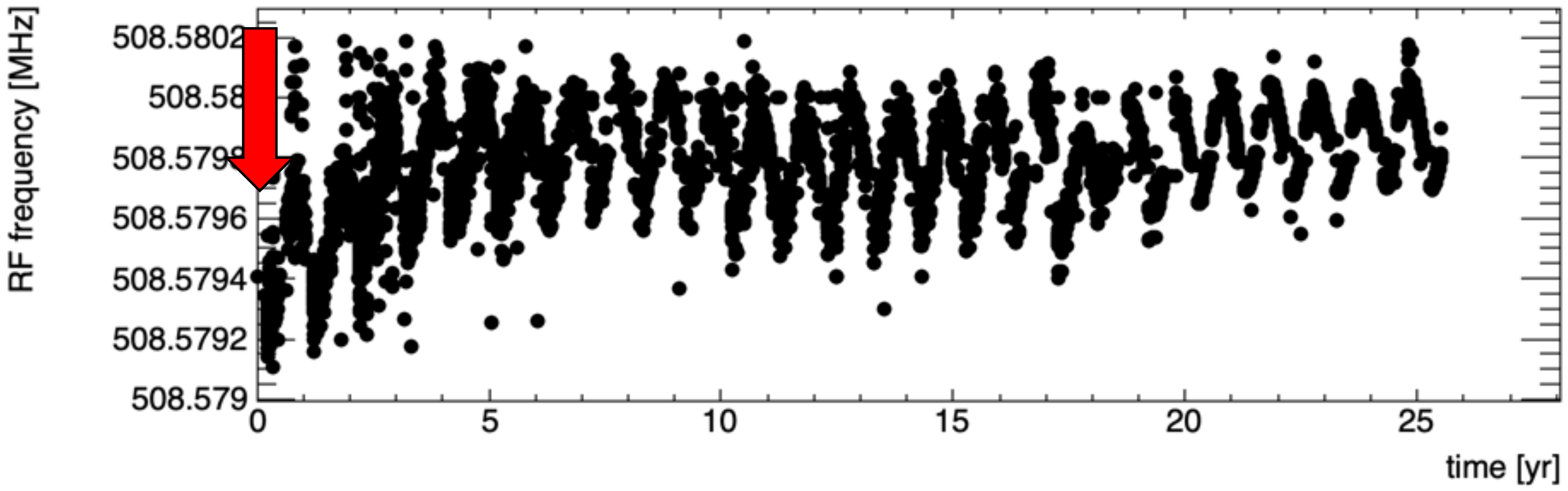
$\Delta p/p \sim 1.08\times10^{-3}$

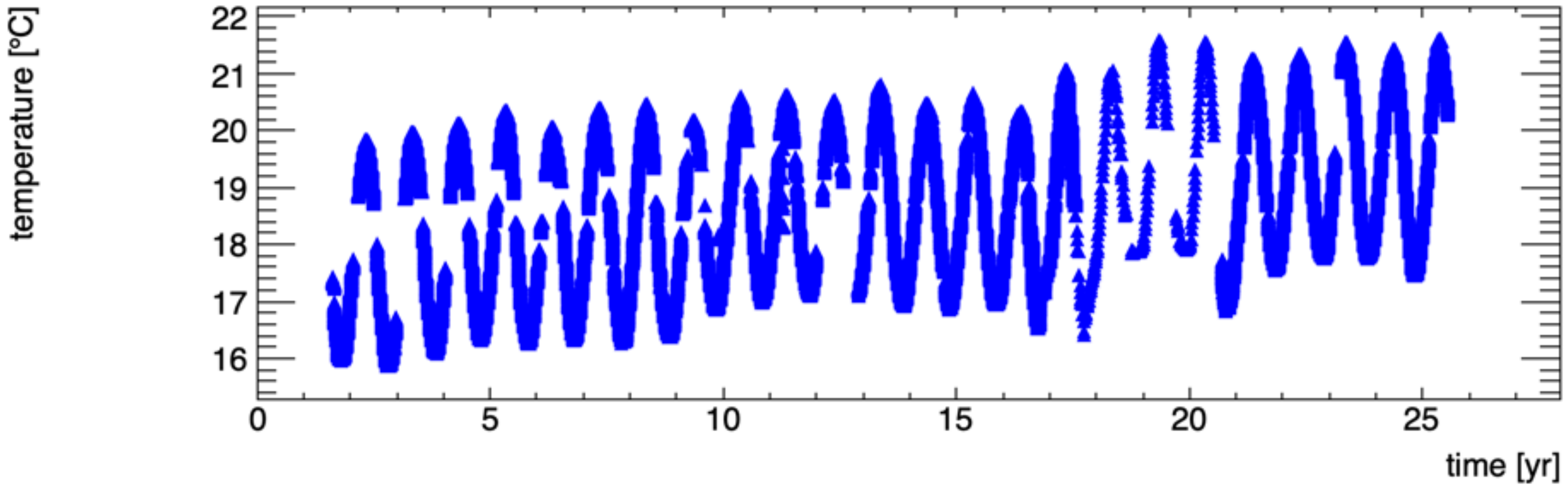
$$\rightarrow \frac{\Delta L}{L} \sim 10^{-7} = 0.1\ \mathrm{ppm}$$

Observations over **25 years**

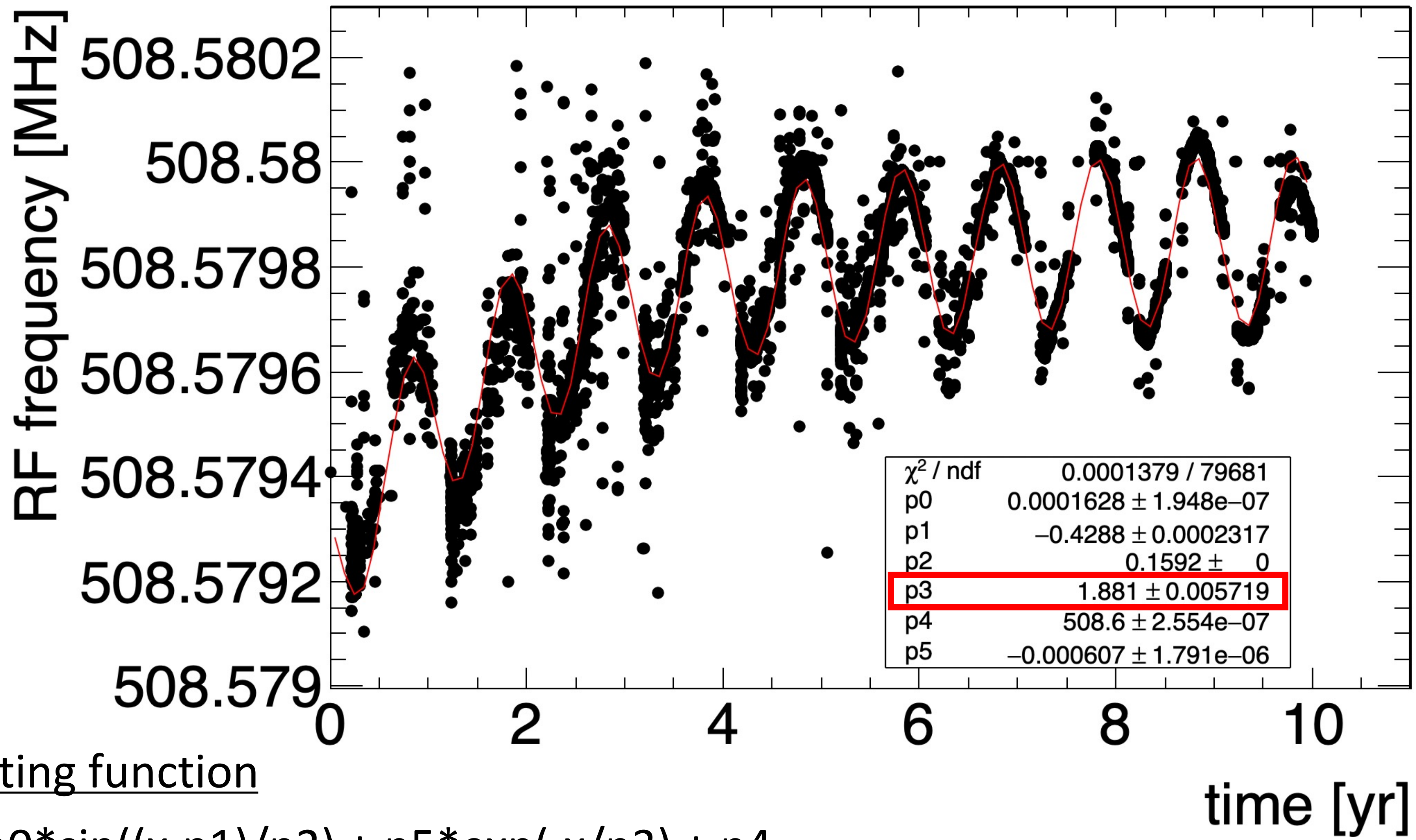
- Annual modulation (0.32 ppm) is clearly correlating to the temperature modulation
- → thermal shrinkage

- Significant shift of RF in the first 2 years cannot be explained by temperature





Time domain analysis: fitting the initial RF shift



The ring shrunk by around 1.2 mm (0.86 ppm) with a time constant of 1.88 years

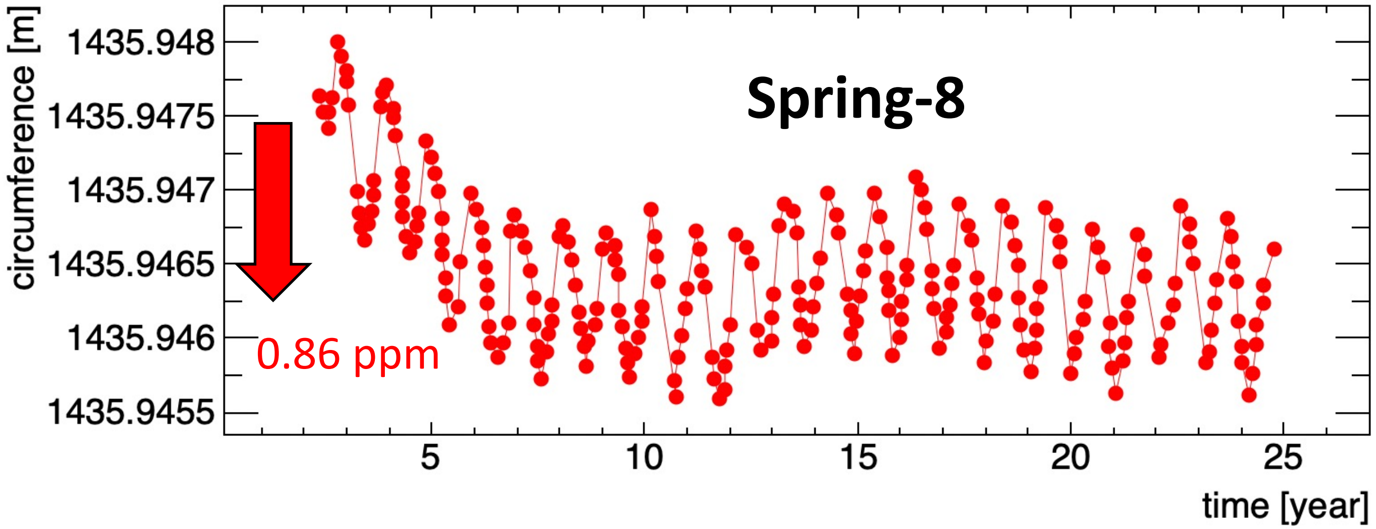
Very good resolution to monitor strain

Fitting function

p0*sin((x-p1)/p2) + p5*exp(-x/p3) + p4

p2 was fixed at 2πx1 yr

Hypothesis: Shrinkage of drying concrete?



→ Compare to a dedicated study of concrete (Japanese paper)

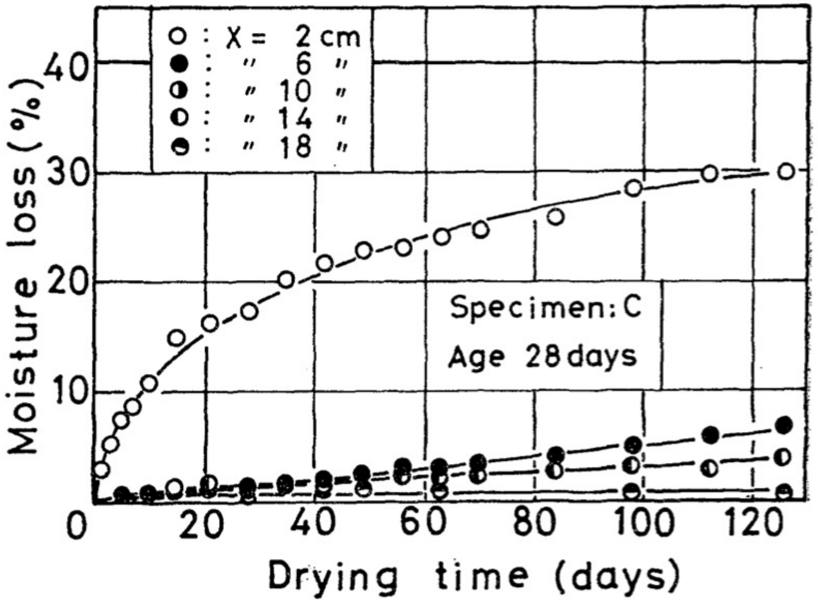
K. Sataka and K. Osamu, A study of the water diffusion and shrinkage in concrete by drying, 土木学会論文報告集 第316号 1981年 12月

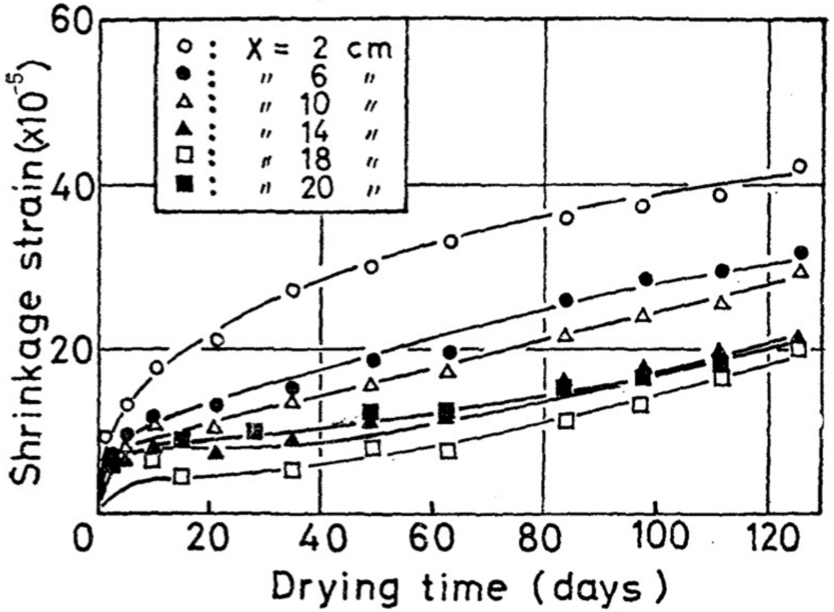
<u>Constant temperature (20C) and humidity (60%) room</u>

X: depth from surface

Shrinkage 10^{-4}/ 100 days if the depth is >20 cm

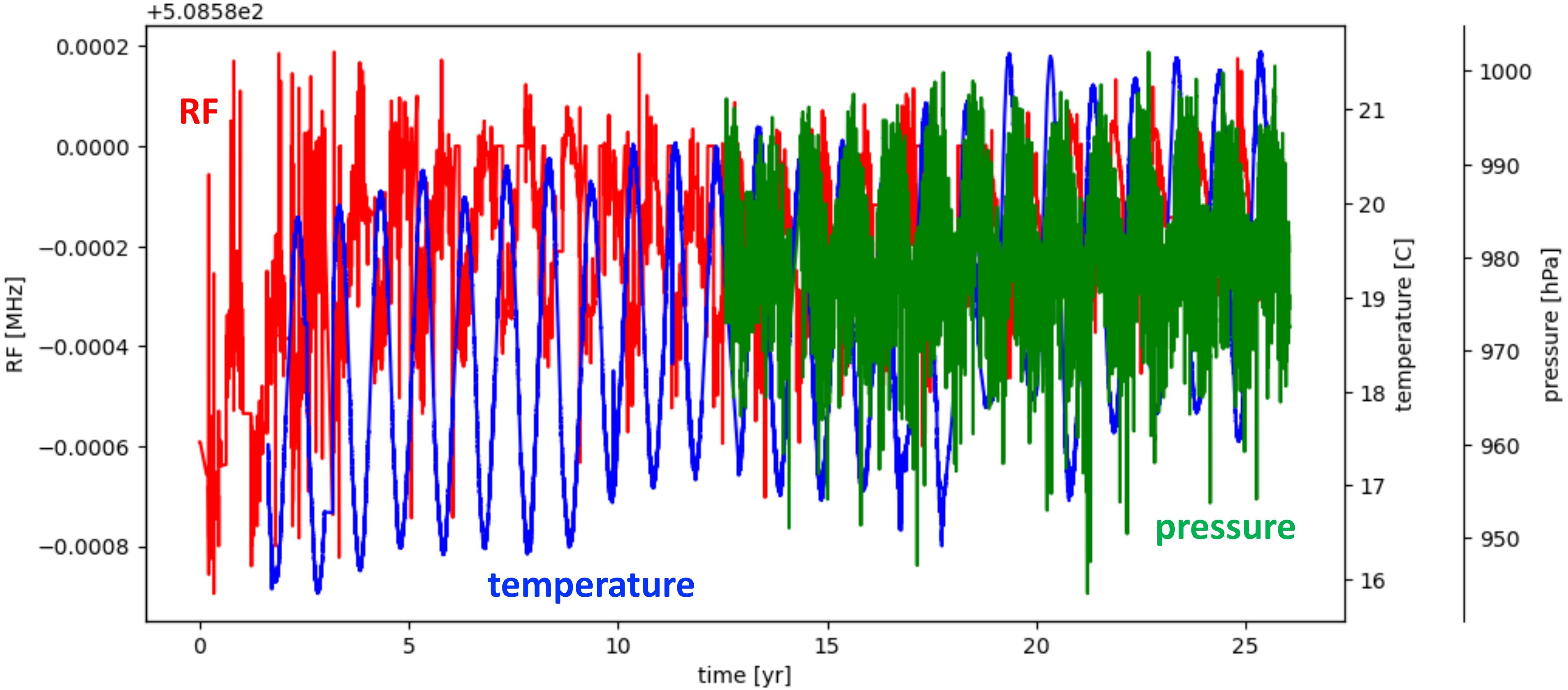
~~SPring-8 is 5 m underground~~





Spectral analysis

Time domain data over 25 years



Power spectrum is of great interest

But the data was not recorded periodically → FFT is not applicable

The Astrophysical Journal Supplement Series, 236:16 (28pp), 2018 May https://doi.org/10.3847/1538-4365/aab766

CrossMark

Understanding the Lomb–Scargle Periodogram

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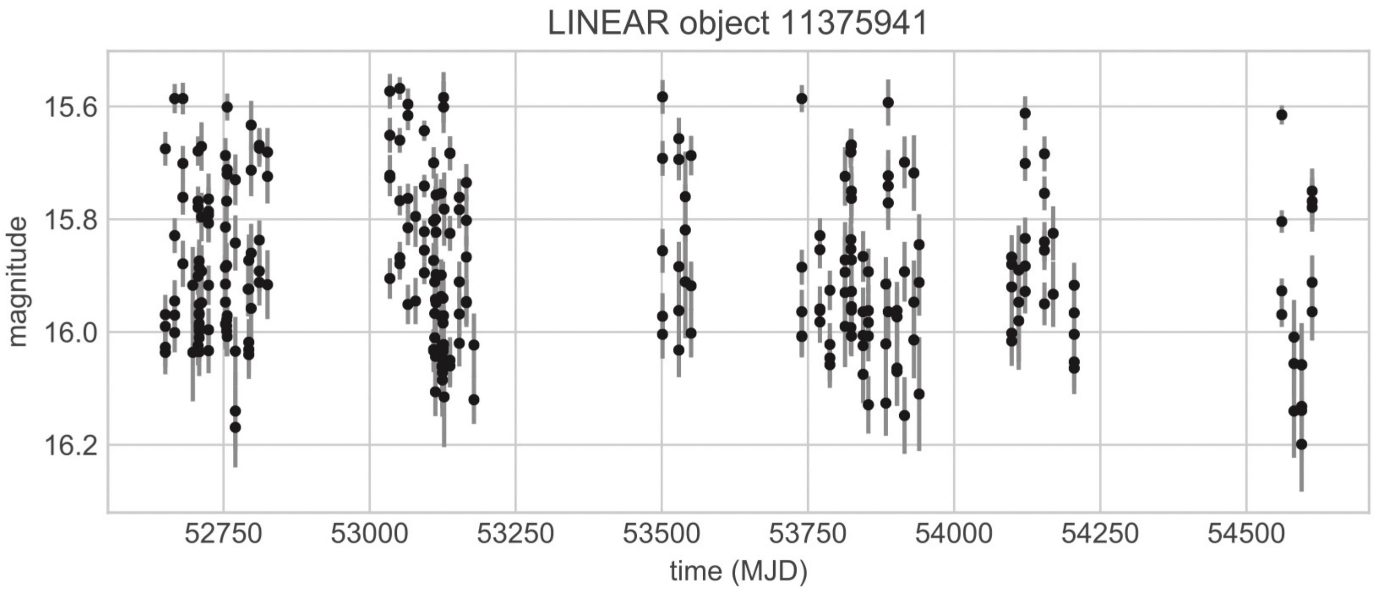
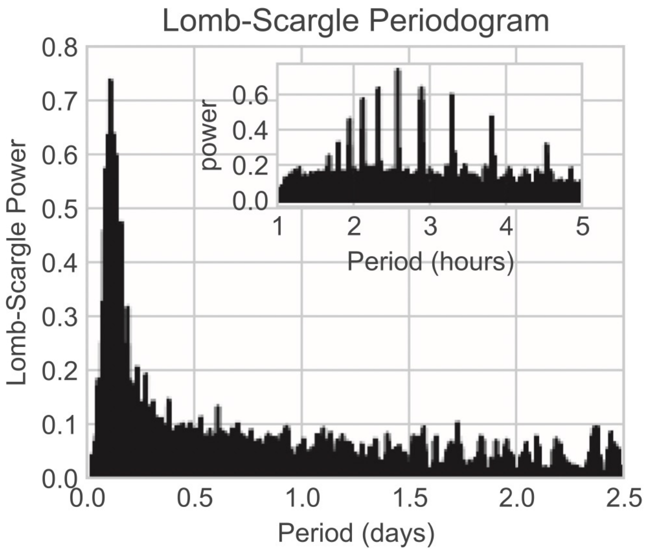
Thanks to Yuto Minami



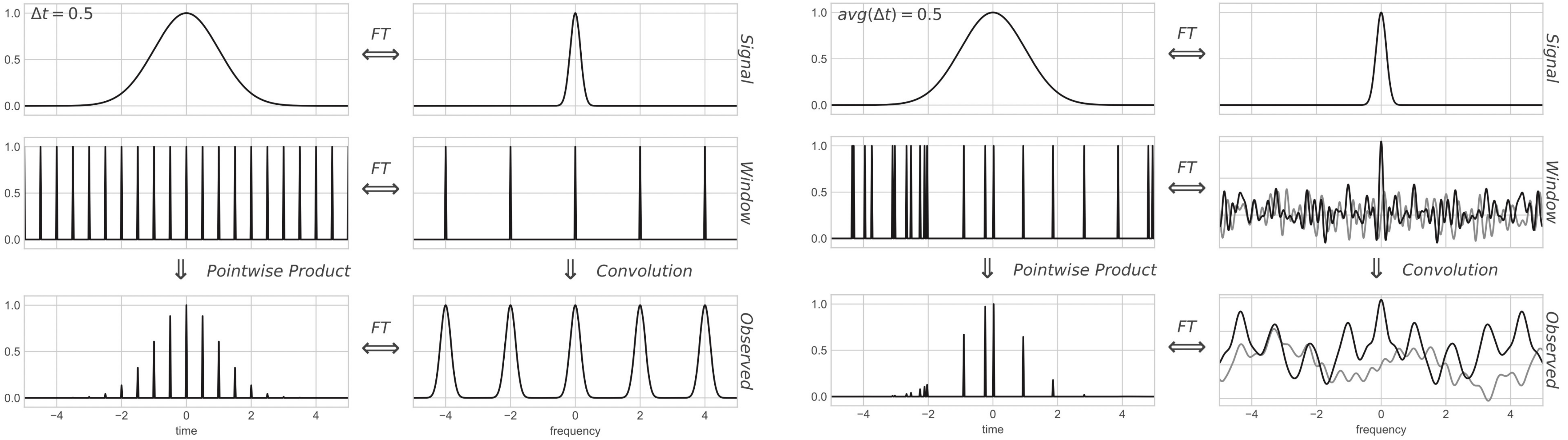


- Commonly used algorithm in the astronomy
- Modified from classical Fourier transform of unevenly spaced data (Schuster periodogram)
- The y-axis is not the classical spectral power density but chi2 of fitting time domain data by a sum of sinusoidal functions

Uneven Fourier analysis: uneven Dirac comb

- Gaussian signal in time domain
- Different recording times

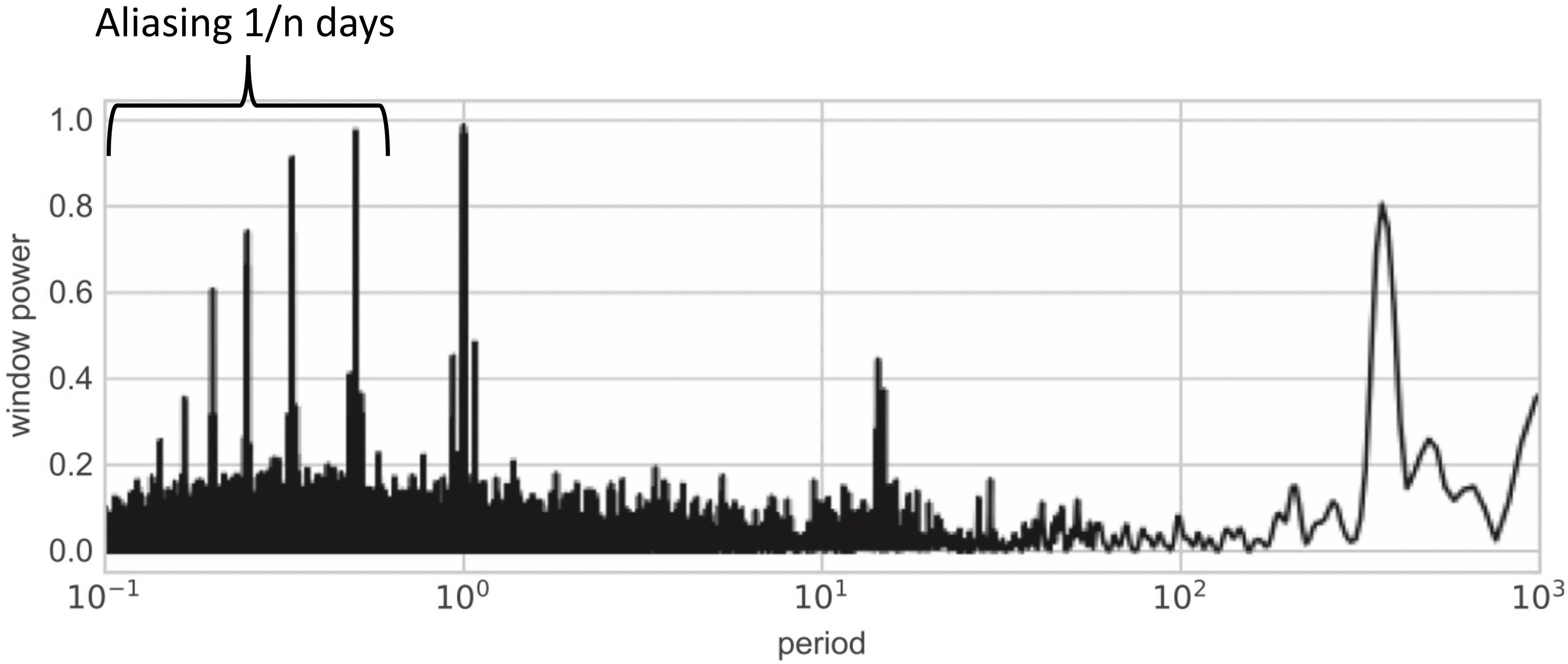
Uniform data acquisition (Dirac comb) | Nonuniform data acquisition



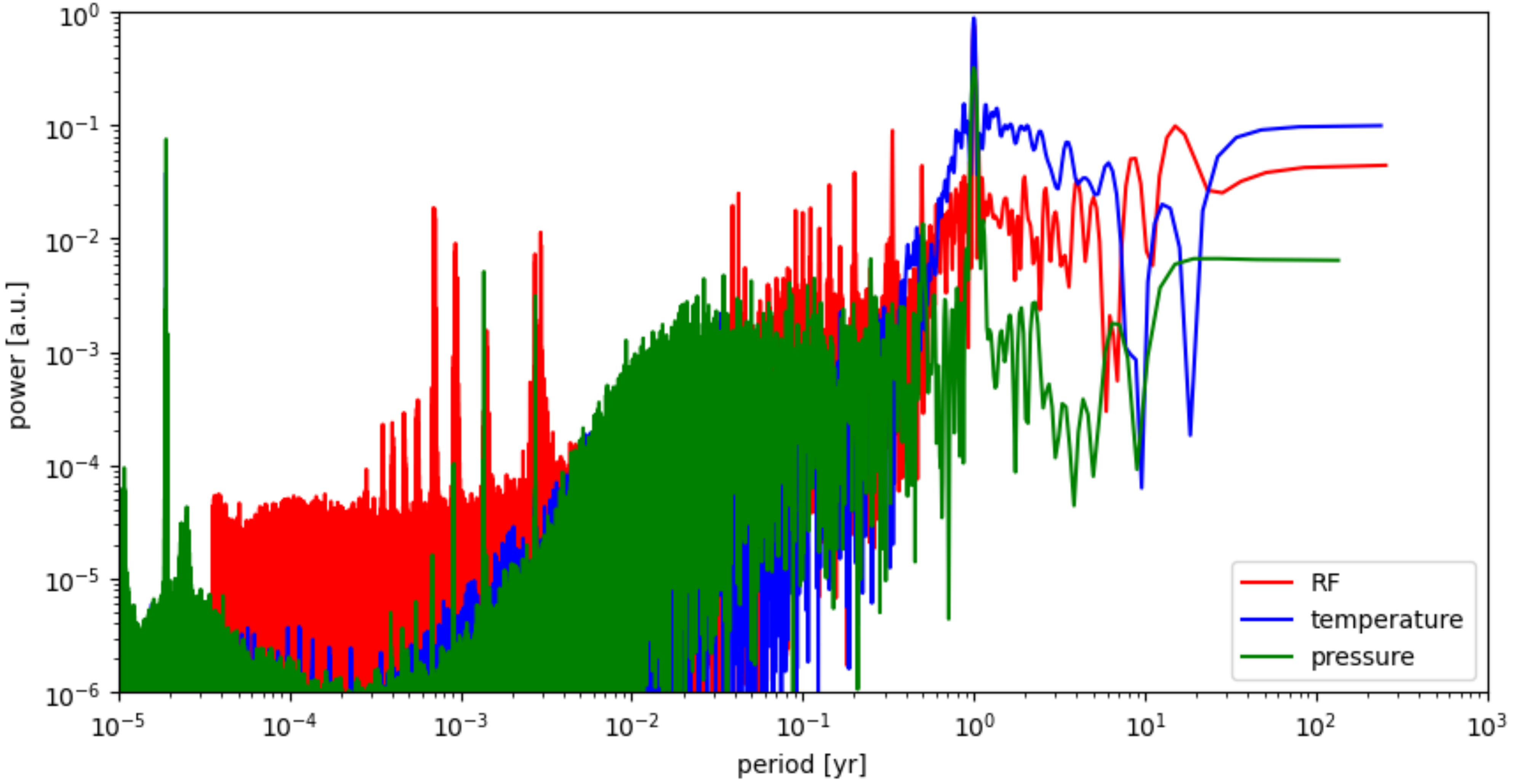
The Fourier spectrum is a convolution of the signal and the window

Fake peaks in unevenly spaced Fourier analysis

- Lomb-Scargle is not free from fake peaks due to aliasing and window
  - Window is non-uniformly distributed Dirac comb & finite period of data set
  - Aliasing is from temporal resolution and different from classical Nyquist for FFT (f/2)



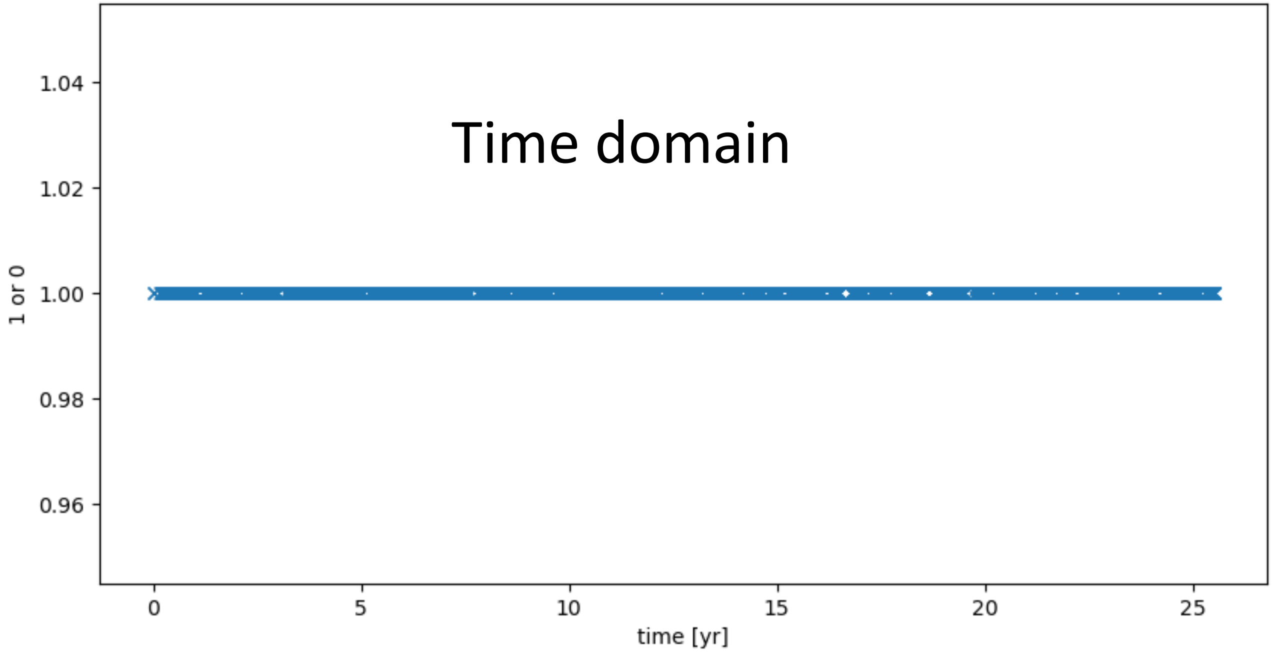
Periodogram of SPring-8 data

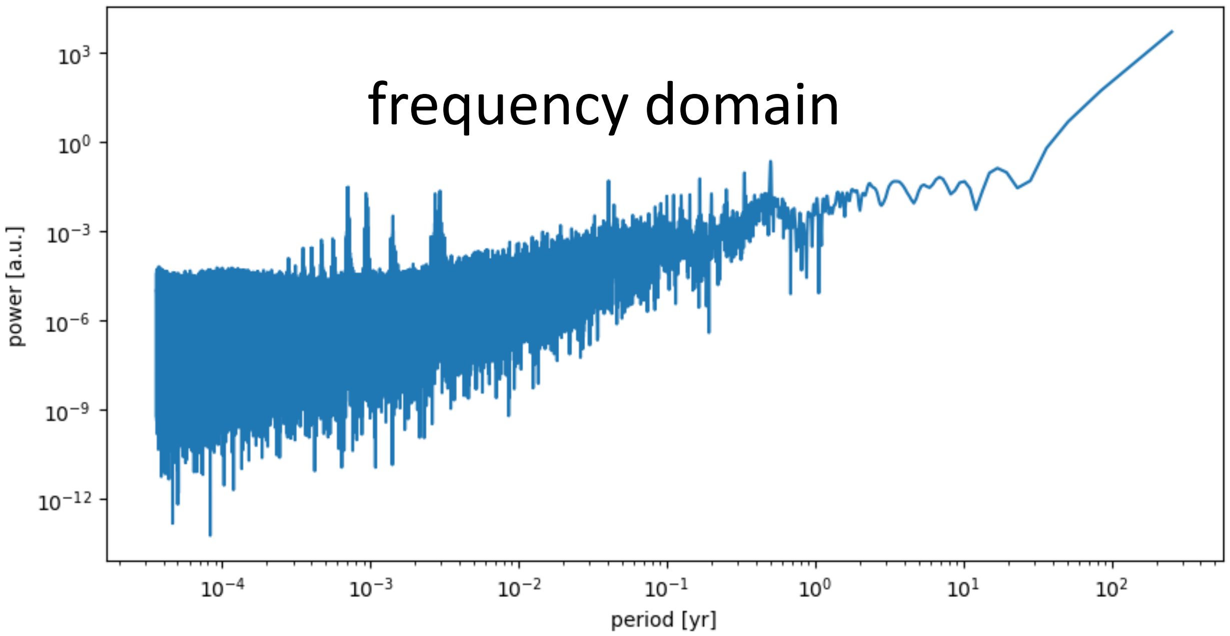


Some peaks are correlated among RF, temperature and pressure data

→ Data were not taken in the same time → time window need to be analyzed

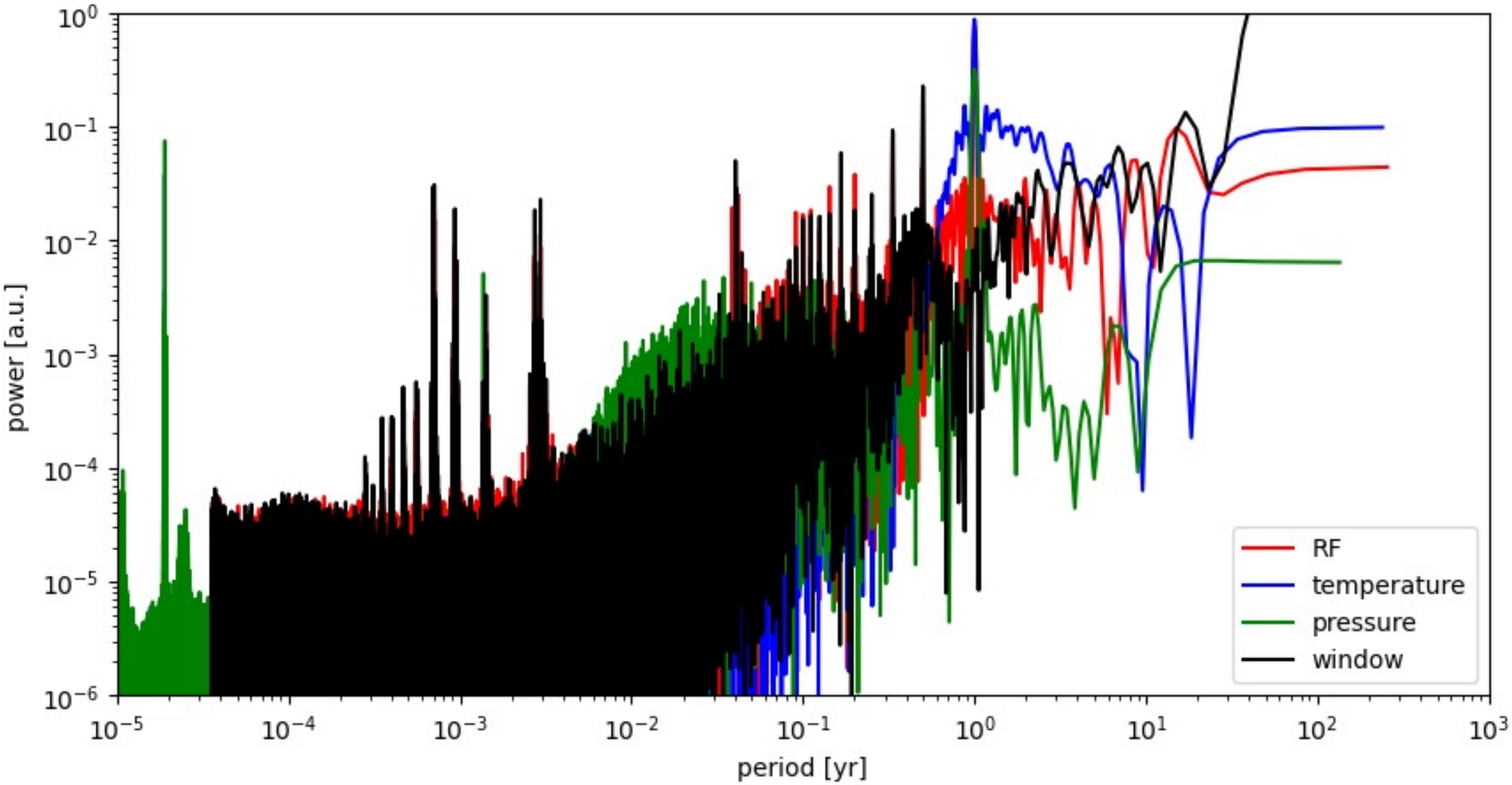
Window function of RF data





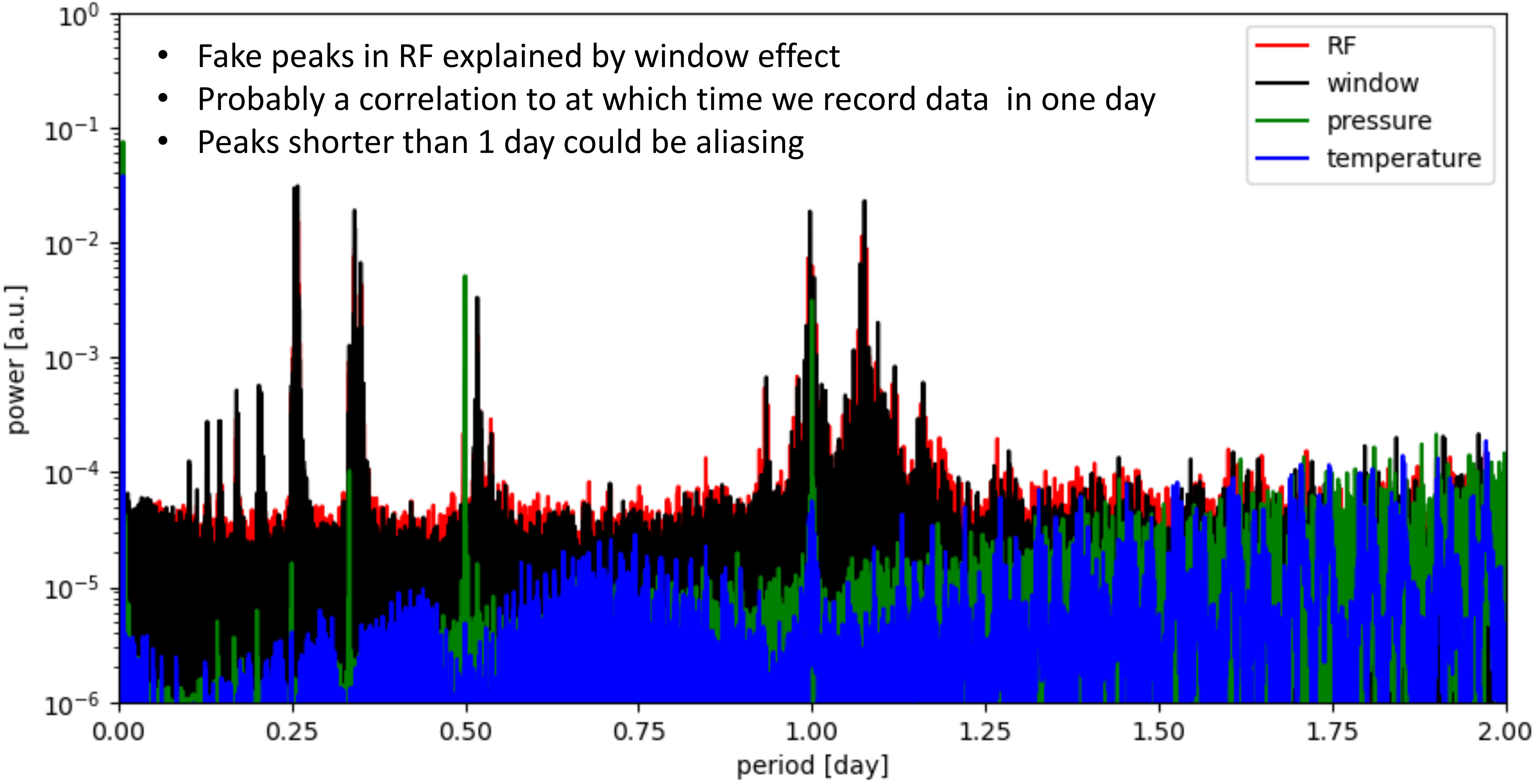
- RF data was not taken during shutdown, RF testing, etc
- Prepare a time-domain data set of ON/OFF
  - Fill "1" on the time when RF data was measured
- Apply Lomb-Scargle to this ON/OFF data set
  - Peaks are clearly visible in frequency domain
  - "Fake" peaks due to periodic shutdown etc

Compare window effect and data (all)

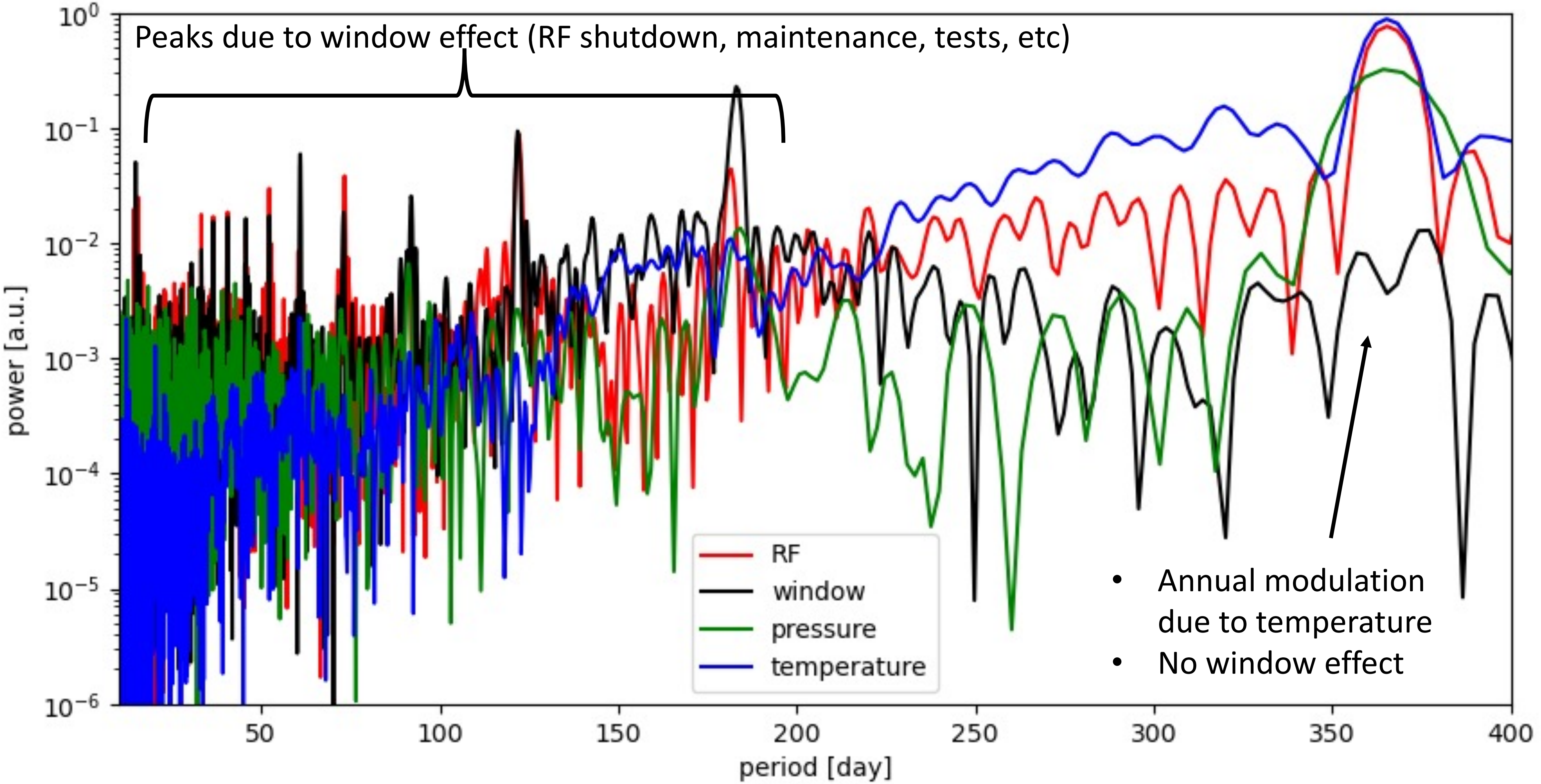


Compare window effect and data (around 1 day)

- Fake peaks in RF explained by window effect
- Probably a correlation to at which time we record data in one day
- Peaks shorter than 1 day could be aliasing

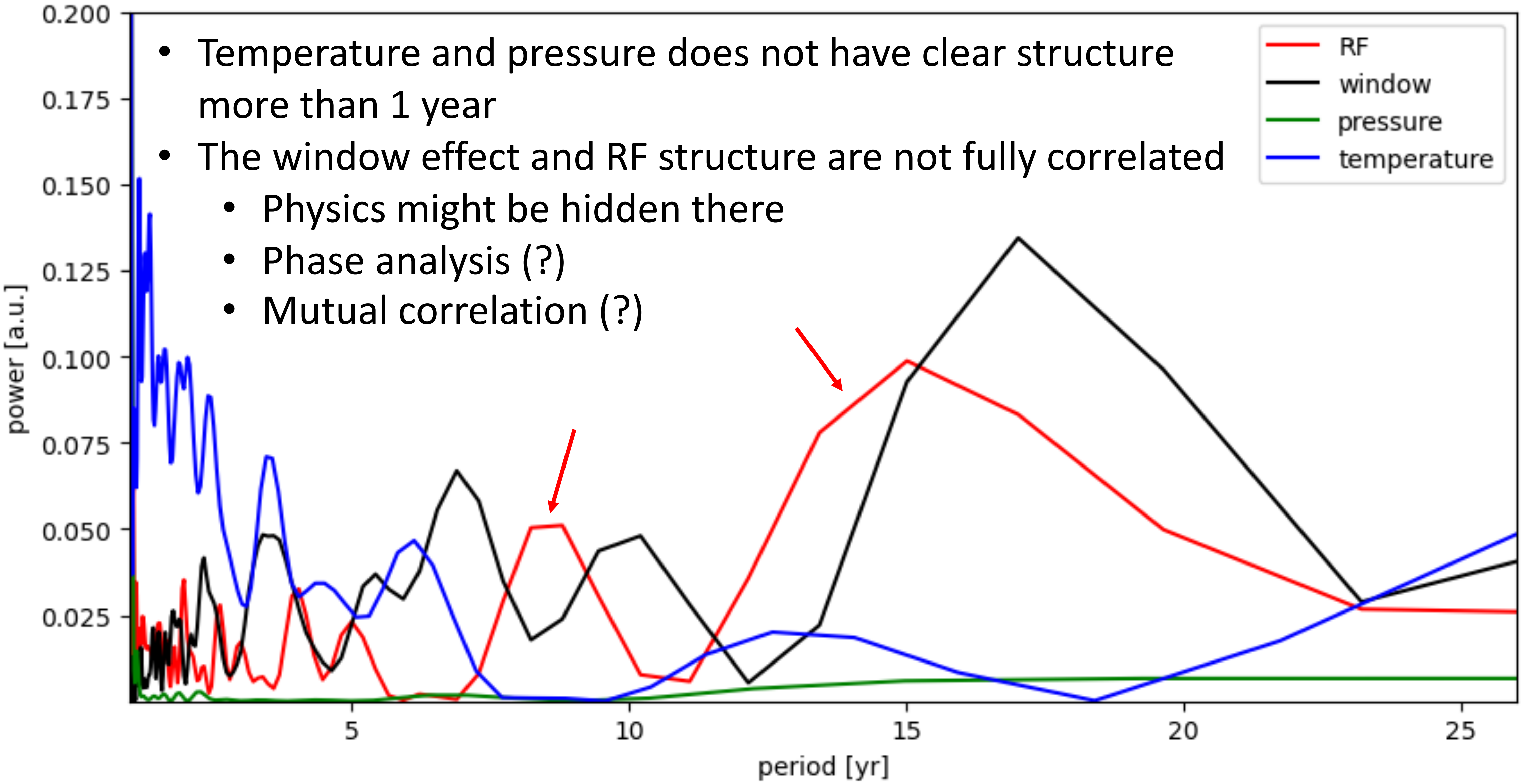


Compare window effect and data (up to 365 days)



Compare window effect and data (years)

- Temperature and pressure does not have clear structure more than 1 year
- The window effect and RF structure are not fully correlated
  - Physics might be hidden there
  - Phase analysis (?)
  - Mutual correlation (?)



Stochastic relic GW (arXiv:gr-qc/0210091)

Stochastic GW

$$\frac{\delta C_{gw}}{C_0} = -\frac{1}{16}\int_0^\infty d\Omega_s \int_0^\infty d\phi \left\langle \left(h_{xx}(t,\theta_s,\phi_s,\omega)\cos 2\phi + h_{xy}(t,\theta_s,\phi_s,\omega)\sin 2\phi\right)^2 \right\rangle_\omega$$

Isotopically coming GW from all solid angle

phase average

Frequency average with spectral density $S_h(\omega)$

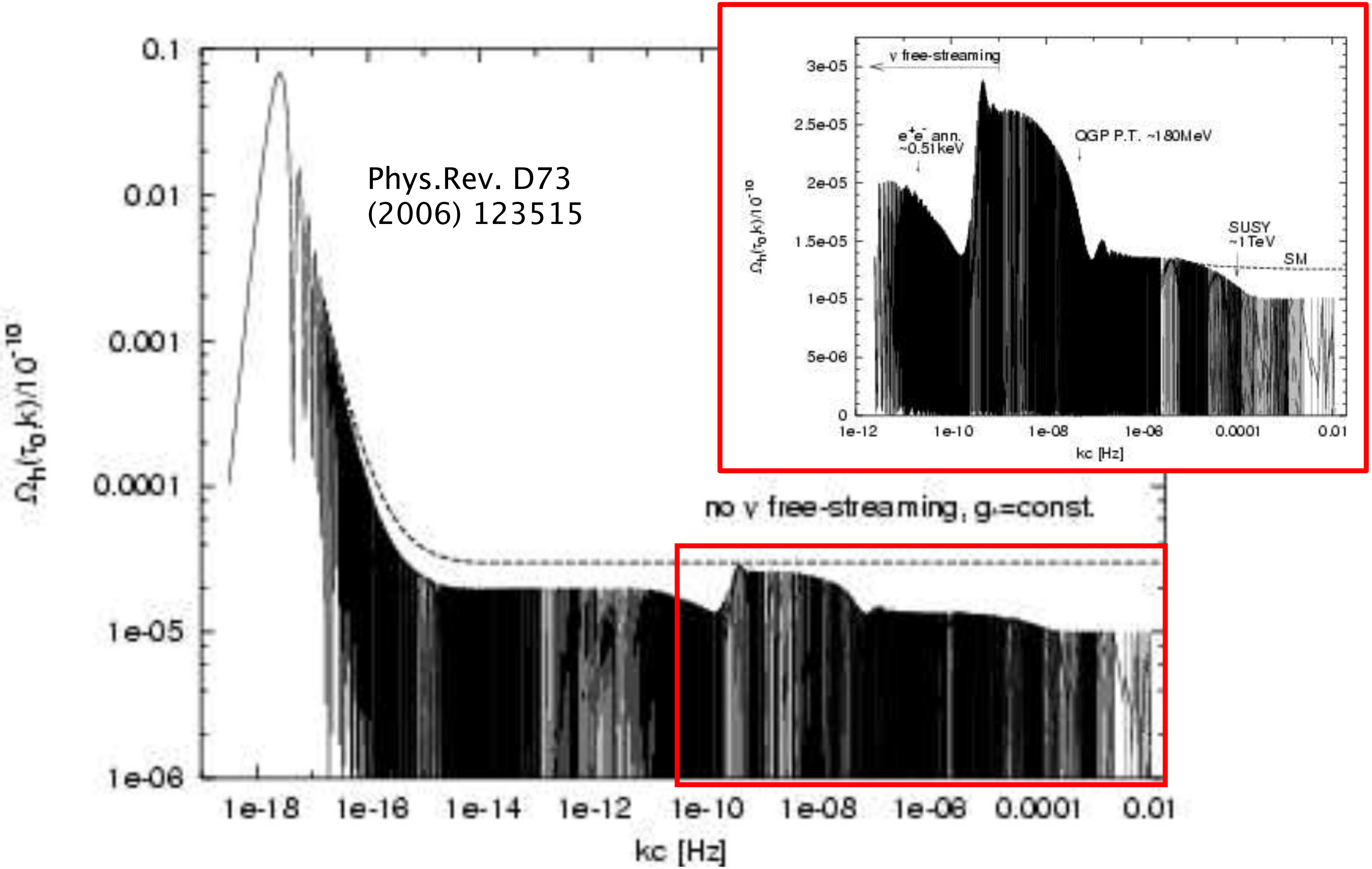
$$\langle f \rangle_\omega = \int_0^\infty d\omega S_h f(\omega)$$

$$\rightarrow \frac{1}{C_0}\frac{\Delta C_{gw}}{\Delta t} = \frac{4\pi}{3}\frac{1}{16}h_0^2 \int_0^\infty \omega S_h(\omega)\sin(2\omega t + 2\delta)\, d\omega$$

$$C_0 \sim T_0 v = \frac{c}{f_{RF}/n}$$

$n = 2436$

$S_h(\omega)$ depends on cosmology and astrophysics



Phys.Rev. D73
(2006) 123515
0.1
0.01
0.001
0.0001
1e-05
1e-06
Ωh(τ0,k)/10^-10
1e-18
1e-16
1e-14
1e-12
1e-10
1e-08
1e-06
0.0001
0.01
kc [Hz]
no ν free-streaming, g*=const.
ν free-streaming
3e-05
2.5e-05
2e-05
1.5e-05
1e-05
5e-06
0
e+e- ann.
~0.51keV
QGP P.T. ~180MeV
SUSY
~1TeV
SM
1e-12
1e-10
1e-08
1e-06
0.0001
0.01
kc [Hz]

Conclusion

- Stochastic GW is very important subject in fundamental physics
  - We are the most interested in something from the very early universe
- NANOGrav published pulsar data of 15 years
- A theory predicts that stochastic GW background would interact with circumference of ring structure in the 2^{nd} order
  - Some issues in the calculation need to be assessed

→ If this is true, storage rings may be a research tool of GWs

- We extracted 25 years data from SPring-8
  - Circumference is precisely reconstructed from RF frequency data and has sub-ppm resolution
  - The annual modulation is correlated to the underground temperature so this is caused by thermal shrinkage
- Time domain analysis revealed exponential shrinkage of the ring
  - 0.86 ppm and time constant of 1.88 years
  - Drying concrete may qualitatively explain this phenomenon but quantitative study seems hard
- Frequency domain spectral analysis revealed interesting structure
  - Lomb-Scargle algorithm was used (common tool in astronomy)
  - Some of the structures were explained by the window bias caused by shutdown, maintenance, DAQ time, etc
  - Some peaks at very low frequency cannot be explained by the window, temperature, and pressure variation
- To which $S(\omega)$ should we compare the data and either support or exclude the theory?
- Can we make a theory-experiment collaboration for data analysis?

backup

Assumptions of relic gravitational waves

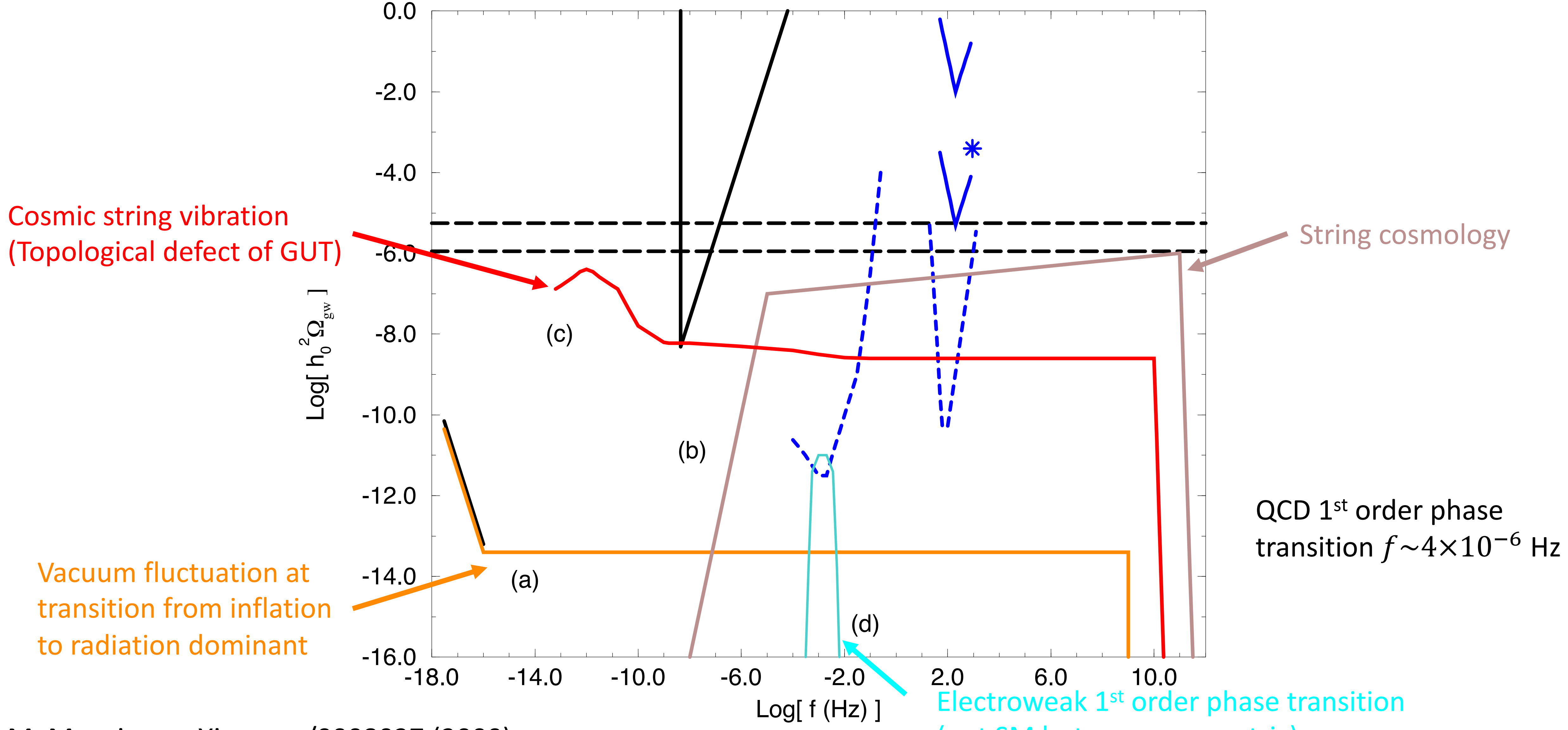
- Decoupling from equilibrium: $(\Gamma/H) = (T/M_{PI})^3 \sim 1 \rightarrow 10^{-44}$ sec
  - Γ: interaction rate, H: Hubble parameter
- Isotropic
- Unpolarized: $\Delta_+ \sim \Delta_\times$
- Stationary
- Intensity of of relic GW: $\Omega_{GW} = \frac{8\pi G_N}{3H_0^2} \frac{d\rho_{GW}}{d \log f}$
  - Hubble constant $H_0 = h_0 \times 100\ \mathrm{km/sec\,MPc}$
- Characteristic amplitude of GW amplitude

$$h_c(f, \Delta f) = 7.111 \times 10^{-22} \left(\frac{1\ \mathrm{mHz}}{f}\right)^{3/2} \left(\frac{h_0 \Omega_{GW}(f)}{10^{-8}}\right)^{1/2} \left(\frac{\Delta f}{3.17 \times 10^{-8}\ \mathrm{Hz}}\right)^{1/2}$$

  - Measurement bandwidth $\Delta f = 1/T$, with T total observation time

M. Maggiore arXiv:gr-qc/0008027 (2000)

Theoretical predictions in 2000 (to be updated)



M. Maggiore arXiv:gr-qc/0008027 (2000)

Remark: relation between h_0^{GW} and h_0

arXiv:gr-qc/0210091

They assumed that the seasonable force is from the gravitational force between the Sun and the Earth:

$$U_s(\boldsymbol{r}) = G_N M_\odot \left(\frac{1}{|\boldsymbol{R}_s - \boldsymbol{r}|} - \frac{1}{R_s} - \frac{\boldsymbol{r} \cdot \boldsymbol{R}_s}{R_s^3} \right)$$

Radius of the Earth $r = 6.378 \times 10^6$ m
Distance between the Sun and the Earch $R_s = 1.496 \times 10^{11}$ m
Mass of the Sun $M_\odot = 1.989 \times 10^{30}$ kg
Newton constant $G_N = 6.638 \times 10^4$ $m^3 kg^{-1} yr^{-2}$

And assumed that energy density of this seasonal force and relic GW is the same due to the same stiffness of the machine

→ They estimated $h_{GW}^0 \sim 5 \times 10^{-16}$ and $h_0 \sim 7 \times 10^{-4}$, so $f = \omega / 2\pi \sim 2 \times 10^{-8} \sim 1.6$ yr and $\Omega_{GW} \sim 5 \times 10^{-11}$

However, Takao's data excludes the assumption

- Motion between the Sun and the Earth must be correlated to the atmospheric temperature
- Takao's data showed that the circumference's change is correlated to the underground (5 m) temperature that significantly delays (2-3 months) from the atmospheric temperature

→ The cause of seasonable change is from thermal expansion related to the underground temperature